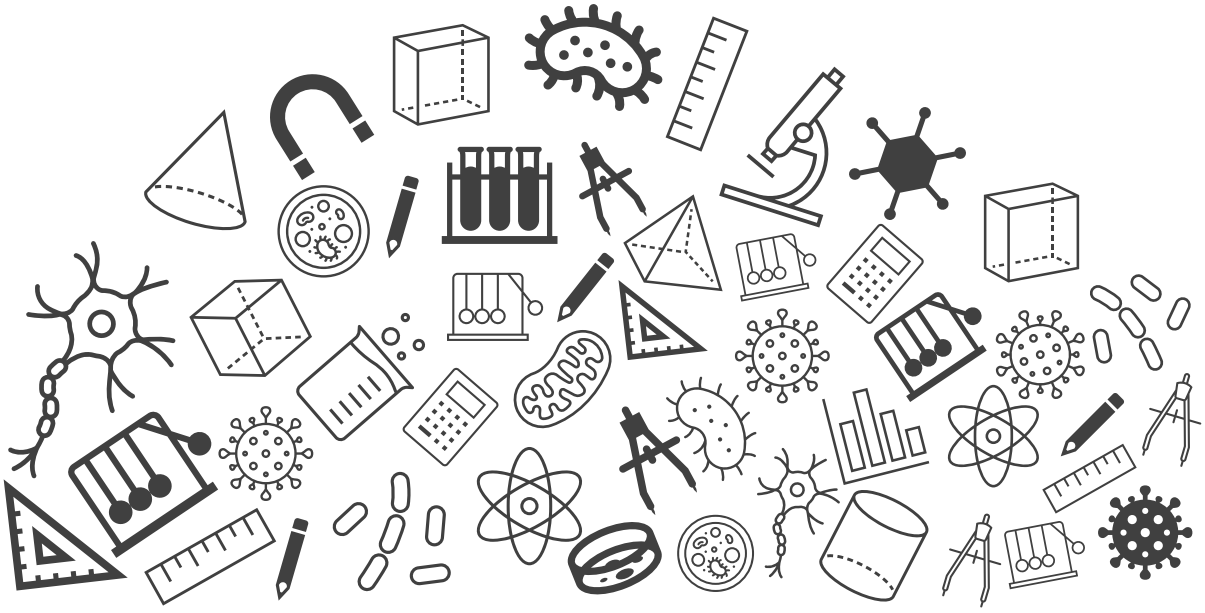
Grade 10

Mathematics

Exam Important Questions



Quadratic Equations

Topic : Exam Important Questions

1. If the roots of the equation $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ are simultaneously real then prove that $b^2 = ac$. [3 marks]

   Given, the roots of both the equations are real.

   First equation:

   $ax^2 + 2bx + c = 0$

   Its discriminant, $D \geq 0$

   $\Rightarrow (2b)^2 - 4(ac) \geq 0$

   $\Rightarrow 4b^2 - 4ac \geq 0$

   $\Rightarrow 4b^2 \geq 4ac$

   $\Rightarrow b^2 \geq ac$ ... (1) (1 Mark)

   Second equation:

   $bx^2 - 2\sqrt{ac}x + b = 0$

   Its discriminant, $D \geq 0$

   $\Rightarrow (2\sqrt{ac})^2 - 4(b^2) \geq 0$

   $\Rightarrow 4ac - 4b^2 \geq 0$

   $\Rightarrow 4ac \geq 4b^2$

   $\Rightarrow ac \geq b^2$... (2) (1 Mark)

   The results of equation (1) and (2) are simultaneously possible in only one case when $b^2 = ac$. (1 Mark)

2. Show that the roots of the equation $x^2 + px - q^2 = 0$ are real for all real values of p and q.
[2 marks]

Given equation is:

$x^2 + px - q^2 = 0$

The discriminant of the given equation is given by

$D = p^2 - 4 \times (1) \times (-q^2)$

$= p^2 + 4q^2$

[1 mark]

Clearly, $D = p^2 + 4q^2 > 0$ for all $p, q \in R$.

Hence, the given equation has real roots.
[1 mark]

3. Find the nature of the roots of the following quadratic equation. If the real roots exist, find them;
$2x^2 - 6x + 3 = 0$ [3 marks]

$2x^2 - 6x + 3 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 2, b = -6, c = 3$
$Discriminant = b^2 - 4ac$
$= (-6)^2 - 4(2)\ (3)$
$= 36 - 24 = 12$
$As\ b^2 - 4ac > 0,$
Distinct real roots exist for this equation.
(1 Mark)

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$

$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$

$= \frac{3 \pm \sqrt{3}}{2}$

$Therefore, the\ roots\ are \frac{3+\sqrt{3}}{2} and \frac{3-\sqrt{3}}{2}$

(2 Marks)

4. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

[3 Marks]

Let the base of the right triangle be $x\ cm$.
Its altitude = $(x-7)\ cm$
From Pythagoras theorem, we have
$Base^2 + Altitude^2 = Hypotenuse^2$
$\therefore x^2 + (x-7)^2 = 13^2$
$\Rightarrow x^2 + x^2 + 49 - 14x = 169$
$\Rightarrow 2x^2 - 14x - 120 = 0$
$\Rightarrow x^2 - 7x - 60 = 0$
$\Rightarrow x^2 - 12x + 5x - 60 = 0$

(Splitting the middle term -7x as -12x+5x because $-12x \times 5x = -60x^2$)
$\Rightarrow x(x-12) + 5(x-12) = 0$
$\Rightarrow (x-12)(x+5) = 0$
Either $x - 12 = 0$ *or* $x + 5 = 0$,
$\Rightarrow x = 12$ *or* $x = -5$ (1 Mark)
Since sides are positive, x can only be 12.
Therefore, the base of the given triangle is $12\ cm$ and the altitude of this triangle will be $(12-7)\ cm = 5\ cm$.
(1 Mark)

5. Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
[3 Mark]

Let's take Rohan's age = $x\ years$
Hence, his mother's age = $x + 26$
3 years from now;
Rohan's age = $x + 3$
Age of Rohan's mother will =$x + 26 + 3 = x + 29$
The product of their ages 3 years from now will be 360.
$(x + 3)(x + 29) = 360$
$\Rightarrow x^2 + 29x + 3x + 87 = 360 \qquad (0.5Mark)$
$\Rightarrow x^2 + 32x + 87 - 360 = 0$
$\Rightarrow x^2 + 32x - 273 = 0$
$\Rightarrow x^2 + 39x - 7x - 273 = 0$
$\Rightarrow x(x + 39) - 7(x + 39) = 0$
$\Rightarrow (x + 39)(x - 7) = 0$
Either $x + 39 = 0 or x - 7 = 0$
$\Rightarrow x = -39\ or\ x = 7 \qquad (2Marks)$
As age cannot be in negative value,
$x = 7$,Rohan's age = $7\ years$
Mother's age = $7 + 26 = 33\ years \qquad (0.5Mark)$

6. Factorize $\sqrt{5}\ x^2$ + 8x + $3\sqrt{5}$ = 0 into the form $\sqrt{5}\ x^2$ + px + qx +$3\sqrt{5}$ = 0.Find the value of p and q.
[2 marks]

Multiply $\sqrt{5}$ with $3\sqrt{5}$ and we get 15

Pairs of 15 are

$1 \times 15 = 15$ sum 1+ 15 = 16

$5 \times 3 = 15$ sum 5 + 3 = 8

So our required pair is 5 and 3
$(1\ Mark)$

$\sqrt{5}\ x^2$ + 8x + $3\sqrt{5}$ = 0

$\sqrt{5}\ x^2$ + 5x + 3x +$3\sqrt{5}$ = 0

So, p = 5 and q = 3.
$(1\ Mark)$

7. Which of the following are not quadratic polynomials?

(i) $3+4x-7x^2$

(ii) $8x^2-15$

(iii) 6x – 15

(iv) $4x^3-3x$

(v) x^2-x+1

[2 Marks]

i) $3+4x-7x^2$ is a polynomial of degree 2, so it is a quadratic polynomial.

ii) $8x^2-15$ is a polynomial of degree 2, so it is a quadratic polynomial.

iii) 6x – 15 is not a polynomial of degree 2, so it is not a quadratic polynomial.

$(1\ Mark)$

iv) $4x^3-3x$ is a polynomial of degree 3, so it is not a quadratic polynomial.

v) x^2-x+1 is a polynomial of degree 2, so it is a quadratic polynomial.

$(1\ Mark)$

8. Check whether the following is a quadratic equation:

$(x-2)(x+1)=(x-1)(x+3)$

[1 Mark]

$(x-2)(x+1)=(x-1)(x+3)$

$\Rightarrow x^2-x-2=x^2+2x-3$

$\Rightarrow 3x-1=0$

It is not of the form $ax^2+bx+c=0$ as a = 0 in this case.

Hence, the given equation is not a quadratic equation.

[1 Mark]

9. Rahul and Shreya were playing with some marbles. Rahul has one more marble than Shreya. The product of the number of marbles they have is 20. Represent it in the form of quadratic equation.

[2 marks]

- Let Shreya has x marbles, and Rahul has $(x+1)$ marbles.
- The product of marbles they have is 20 i.e., $x(x+1) = 20$
- $x^2+x = 20$ (Quadratic polynomial)
  Bring constant term *in LHS*
  $x^2+x-20 = 0$
  This is the required quadratic equation.

[2 marks]

10. Find the roots of the quadratic equation by applying the quadratic formula.
$4x^2 + 4\sqrt{3}x + 3 = 0$ [3 marks]

$4x^2 + 4\sqrt{3}x + 3 = 0$
On comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 4, b = 4\sqrt{3}\ and\ c = 3$
By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2-4\times4\times3}}{2\times4}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{48-48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{0}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3}}{8}$$

$$\therefore x = -\frac{\sqrt{3}}{2} \qquad (3Marks)$$

11. Find the value of x in $2x^2 - 4x + 2 = 0$ using the quadratic formula.

[2 Marks]

Given the quadratic equation
$2x^2 - 4x + 2 = 0$

We know that,
Standard form of the quadratic equation is $ax^2 + bx + c = 0$

Comparing both we get $a = 2b = -4$ and $c = 2$.

The discriminant is given by:
$D = b^2 - 4ac$
$= 4^2 - 4 \times 2 \times 2$
$= 0$
[1 Mark]
The solutions are found using the formula:
$$x = \frac{-b \pm \sqrt{D}}{2a}$$
$$x = \frac{4 \pm 0}{2 \times 2}$$

On solving
$x = 1$ and $x = 1$
Hence, the value of $x = 1$.
[1 Mark]

12. Find the roots of the quadratic equation by using the quadratic formula:

$$2x^2 - 3x - 5 = 0$$

[2 Marks]

The quadratic formula for finding the roots of quadratic equation $ax^2 + bx + c = 0, a \neq 0$ is given by, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

[1 Mark]

Given, $2x^2 - 3x - 5 = 0$

$$\therefore \mathrm{x} = \frac{-(-3) \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{49}}{4}$$

$$= \frac{3 \pm 7}{4} = \frac{5}{2}, -1$$

[1 Mark]