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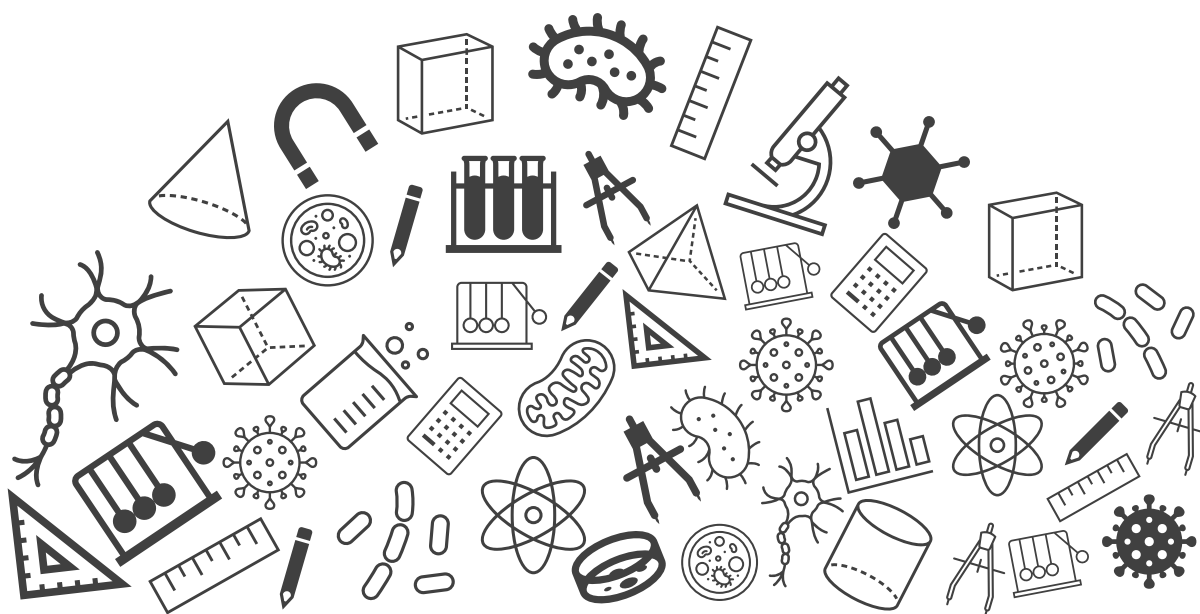
Grade 10
Mathematics
Exam Important Questions

Arithmetic Progressions: Sum of n Terms of an A.P - 2

Topic : Exam Important Questions

1. In an agriculture field, there are 23 cotton plants in the first row, 21 in the second row, 19 in the third row, and so on. There are 5 cotton plants in the last row. How many rows are there in the agriculture field.

   [2 Marks]

   Solution:
   The number of cotton plants in the 1st, 2nd, 3rd, . . ., rows are:

   $23, 21, 19, \ldots, 5$

   It forms an AP as $d = 21 - 23 = 19 - 21 = -2$
   Let the number of rows in the agriculture field be n.

   Then $a = 23$, $d = -2$, $a_n = 5$
   [1 Mark]

   As, $a_n = a + (n-1)d$

   We have, $5 = 23 + (n-1)(-2)$

   i.e., $-18 = (n-1)(-2)$

   i.e., $n = 10$

   So, there are 10 rows in the agriculture field.
   [1 Mark]

2. For the following A.Ps, write the first term and the common difference.
   3, 1, - 1, - 3 …
   [2 marks]

   3, 1, - 1, - 3 …
   Here, first term, a = 3
   [1 mark]
   Common difference, d = Second term - First term
   = 1 - 3 = - 2
   [1 mark]

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3. (a). In an A.P. if the sum of third and seventh term is zero, find its 5th term.

OR

(b). Determine the A.P. whose third term is 5 and seventh term is 9.

[2 Marks]

Solution:

Given, sum of third and seventh term of A.P. is zero.
We know that, nth term of an A.P. is $\mathbf{T_n = a + (n-1)d}$
$\therefore \mathbf{T_3 + T_7 = 0}$
$\Rightarrow \mathbf{a + 2d + a + 6d = 0}$
$\Rightarrow \mathbf{2a + 8d = 0}$
$\Rightarrow \mathbf{a + 4d = 0} \dots (i)$

[1 Mark]

Now,
$\mathbf{T_5 = a + (5-1)d}$
$= \mathbf{a + 4d}$
$= \mathbf{0}(from\ eq(i))$

[1 Mark]

OR

(b).   We have
$a_3 = a + (3-1)d = a + 2d = 5...(i)$

and $a_7 = a + (7-1)d = a + 6d = 9...(ii)$

[0.5 Marks]

Solving the pair of linear equations (i) and (ii):
From equation$(i)-$ equation(ii), we get, $a + 2d - a - 6d = 5 - 9$
$$-4d = -4$$
$$\Rightarrow d = 1$$
[0.5 Marks]

Substitute $d = 1$ in equation(i), we get
$\Rightarrow a + 2(1) = 5$
$\Rightarrow a = 5 - 2$
$\therefore a = 3$

Thus, $a = 3$, $d = 1$

Hence, the required AP is $3, 4, 5, 6, 7 \dots$

[1 Mark]

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4. Identify the missing number

    7, 5, 2, 14, 3, 1, 2, 6, 8, 5, 3, 16, 6, 2, 4, (?).

    [2 Marks]

    7, 5, 2, 14 = 7 + 5 + 2 = 14

    3, 1, 2, 6 = 3 + 1 + 2 = 6

    8, 5, 3, 16 = 8 + 5 + 3 = 16
    [1 Mark]

    6, 2, 4, (?) = 6 + 2 + 4 = 12

    Answer: 12
    [1 Mark]

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5. Show that $a_1, a_2 \ldots \ldots, a_n, \ldots \ldots$ form an AP where a_n is defined as below:

$a_n = 3 + 4n$

Also find the sum of the first 15 terms. ....(3 Marks)

Given, $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

$\qquad (0.5 mark)$

It can be observed that;

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

$\qquad (0.5 mark)$

i.e., $a_{k+1} - a_k$ *is same everytime. Therefore, this is an AP*

with common difference as 4 and first term as 7.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2(7) + (15 - 1) \times 4]$

$= \frac{15}{2}[(14) + 56]$

$= \frac{15}{2}(70)$

$= 15 \times 35$

$= 525$

$\qquad (2 marks)$

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6. In a nursery, 37 plants have been arranged in the first row, 35 in the second, 33 in the third and so on. If there are 5 plants in last row, how many plants are there in the nursery? (2 marks)

   The given sequence is: 37, 35, 33 ...., 5

   $a = 37, \ d = -2, \ a_n = 5$

   $a_n = a + (n-1)d$

   $\Rightarrow 5 = 37 + ((n-1) \times -2)$

   $\Rightarrow 5 = 37 + 2 - 2n$

   $\Rightarrow 5 = 39 - 2n$

   $\Rightarrow 2n = 34$

   $\Rightarrow n = 17$ (1 mark)

   $S_n = \frac{n}{2}(a + a_n)$

   $\Rightarrow \frac{17}{2}(37 + 5)$

   $\Rightarrow \frac{17}{2} \times 42$

   $= 17 \times 21$

   $= 357$ (1 mark)

7. Find the sum of the following APs.

(i) 2, 7, 12,…., to 10 terms

(ii) − 37, − 33, − 29 ,…, to 12 terms …..(3 marks)

(i) $2, 7, 12, \ldots \ldots, to\ 10\ terms$

$For\ this\ A.P.,$

$a = 2$

$d = a_2 - a_1 = 7 - 2 = 5$

$n = 10$

$We\ know\ that,$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(2) + (10-1) \times 5]$

$= 5[4 + (9) \times (5)]$

$= 5 \times 49 = 245$

(1.5 marks)

(ii) $-37, -33, -29, \ldots \ldots, to\ 12\ terms$

$For\ this\ A.P.,$

$a = -37$

$d = a_2 - a_1 = (-33) - (-37)$

$= -33 + 37 = 4$

$n = 12$

$We\ know\ that,$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2(-37) + (12-1) \times 4]$

$= 6[-74 + 11 \times 4]$

$= 6[-74 + 44]$

$= 6(-30) = -180$

(1.5 marks)

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8.

The 10th term of an AP is 52 and 16th term is 82. Find the 32nd term and the general term. (3 marks)

Let a be the first term and d be the common difference of the given A.P. Let the A.P. be $a_1, a_2, a_3..... a_n.....$

It is given that

a_{10} = 52 and a_{16} = 82

$a + (10-1)d = 52$ and $a + (16-1)d = 82$

$a + 9d = 52$ .... (i)

and,

$a + 15d = 82$ .... (ii)

Subtracting equation (ii) from equation (i), we get

$-6d = -30$

$d = 5$ (1.5 marks)

Putting $d = 5$ in equation (i), we get

$a + 45 = 52$

$a = 7$

$a_{32} = a + (32-1)d = 7 + 31 \times 5 = 162$

and,

$a_n = a + (n-1)d = 7(n-1) \times 5 = 5n + 2.$

Hence $a_{32} = 162$ and $a_n = 5n + 2.$ (1.5 marks)

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