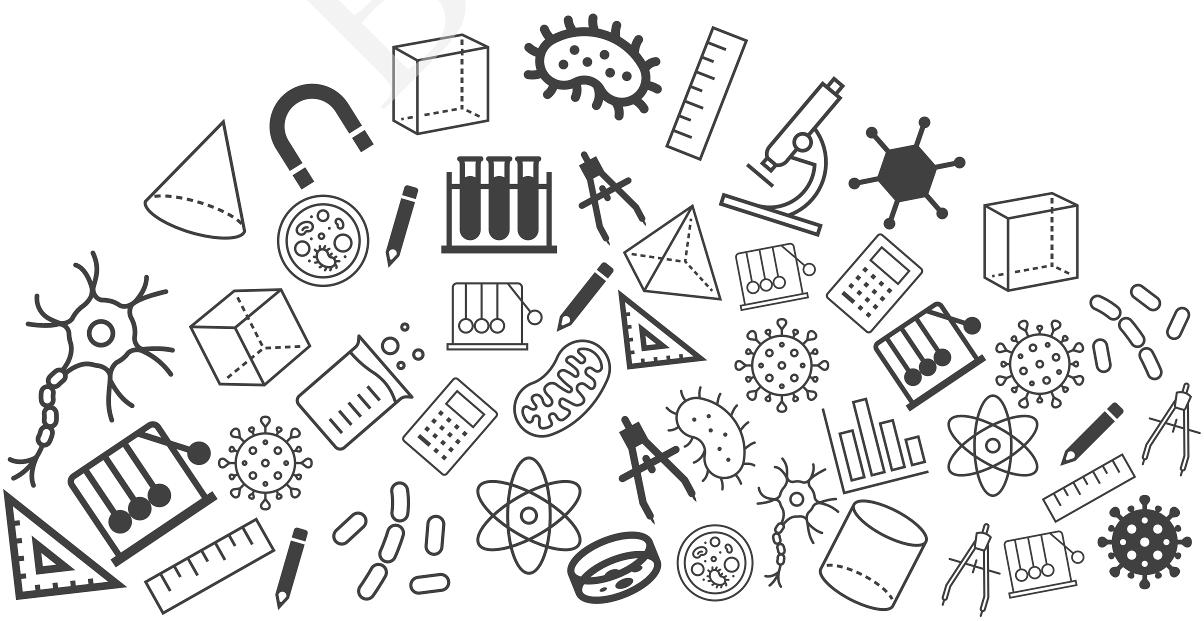
Grade 10
Mathematics Chapter Notes



CONTENTS

MATHEMATICS

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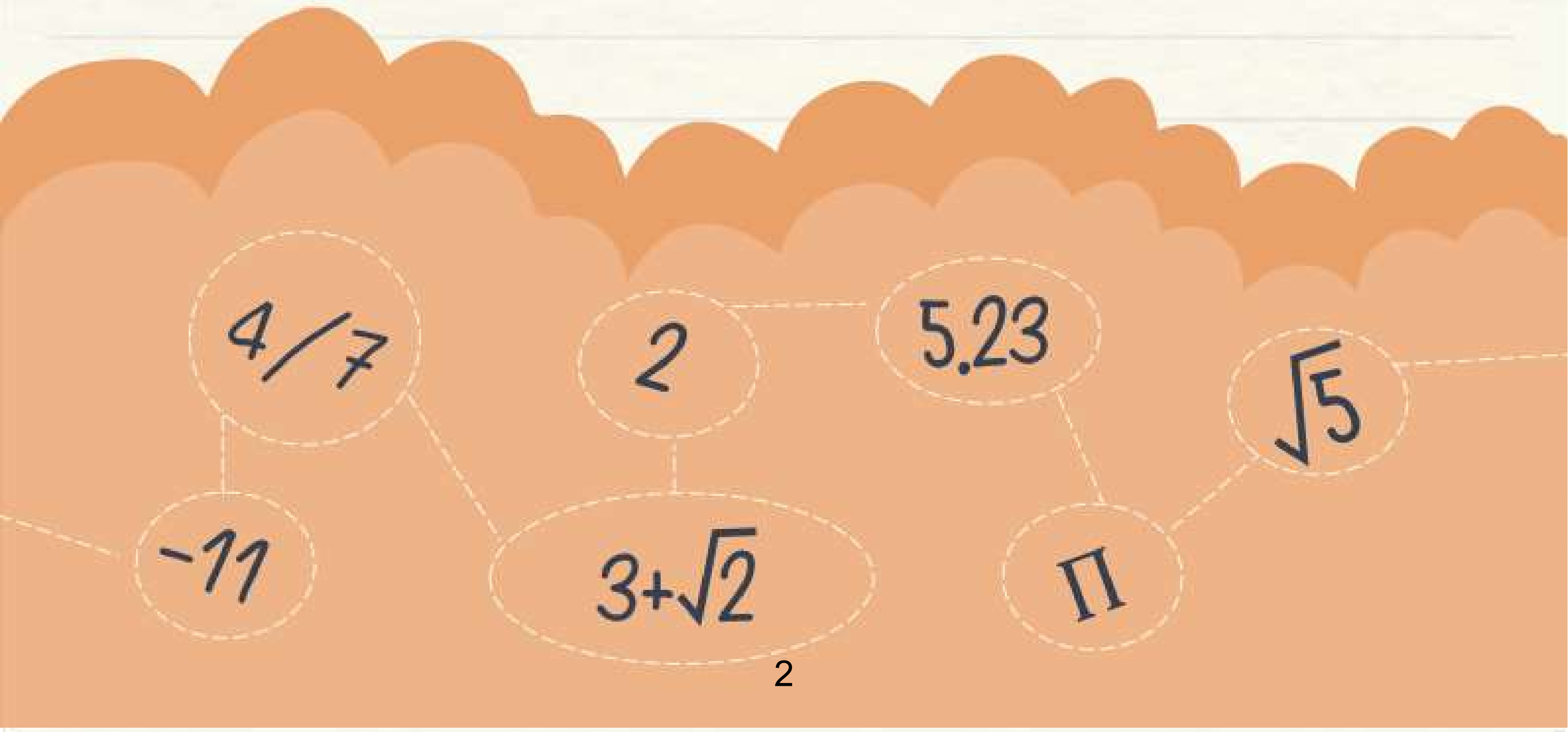
CHAPTER NOTES

Real Numbers

Topics

1. Fundamental Theorem of Arithmetic

2. Irrational Numbers



1. Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique (apart from the order).

Example

The prime factorisation of the number 8190 is:

$$8190 = 2^1 \times 3^2 \times 5^1 \times 7^1 \times 13^1$$

4.1 Theorem Based on Fundamental Theorem of Arithmetic

If a prime number p divides a^2, then p divides a, where a is a positive integer.

Example

Let us consider, p = 3, a = 9

3 divides 9^2

3 divides 9.

4.2 Relation between HCF and LCM

For any two positive integers **a** and **b**,

$$HCF\ (a, b) \times LCM\ (a, b) = a \times b$$

Note: This relationship only holds good for two numbers.

2. Irrational Numbers

A number 's' is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Example

Prove that, $\sqrt{2}$ is irrational.

Proof: By using method of contradiction

Assume $\sqrt{2}$ is a rational number.

$\sqrt{2} = \frac{a}{b}$ (a and b are co-primes and $b \neq 0$)

$\Rightarrow b\sqrt{2} = a$

Squaring both the sides

$(b\sqrt{2})^2 = a^2$

$\Rightarrow 2b^2 = a^2$ (a is an even number)

Let a = 2k (k is an integer)

$2b^2 = 4k^2$

$\Rightarrow b^2 = 2k^2$ (b is an even number)

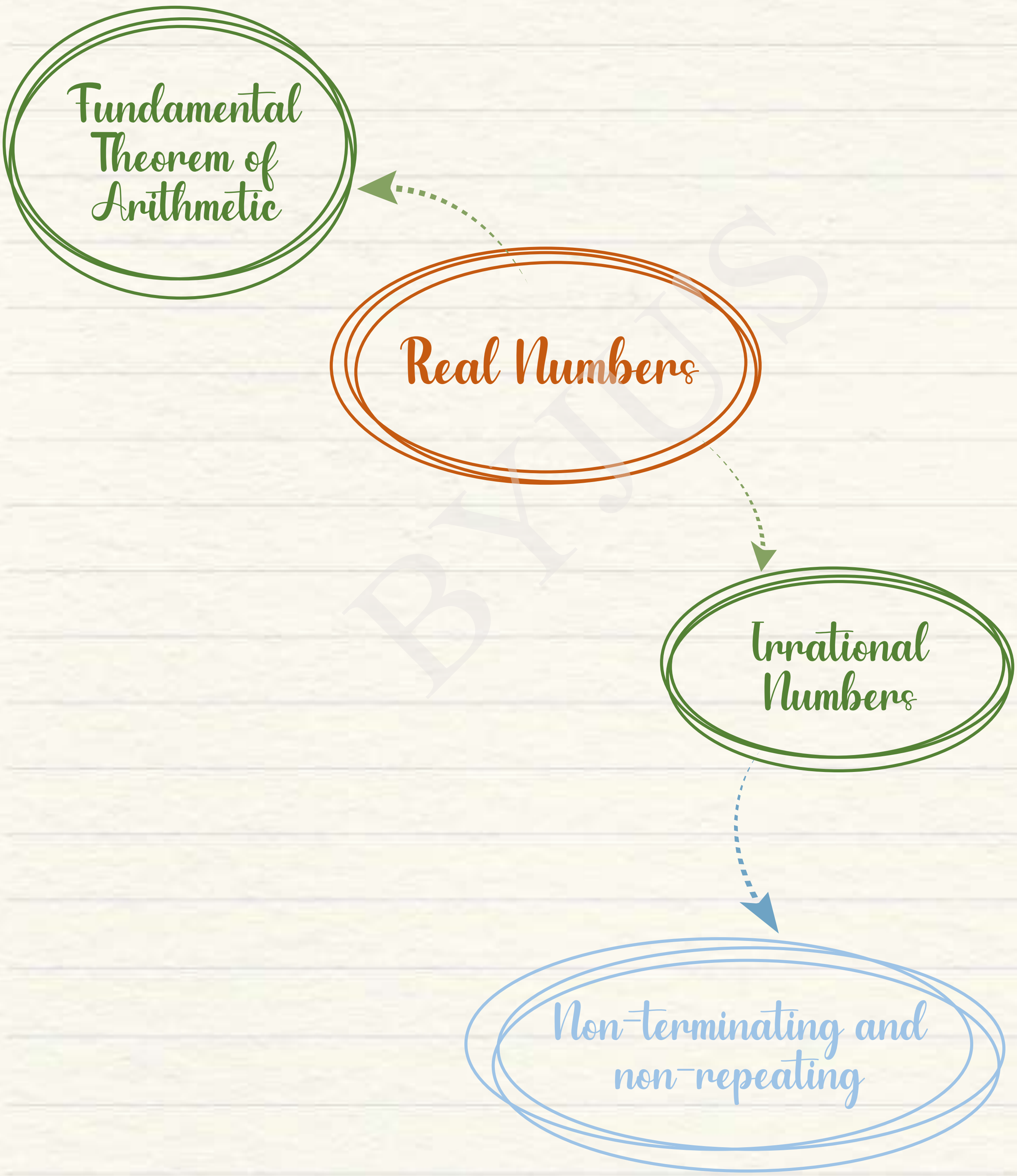
$\therefore$ a and b have 2 as a common factor.

But this contradicts the fact that a and b are co-primes.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

Hence $\sqrt{2}$ is irrational.

Mind Map


Fundamental Theorem of Arithmetic
Real Numbers
Irrational Numbers
Non-terminating and non-repeating

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CHAPTER NOTES

Polynomials

Topics

1. Polynomials and terms related to it
2. Special Types of Polynomials
3. Value of a Polynomial at a Point
4. Zeroes of a Polynomial
5. Relationship between Zeroes and Coefficients of a Polynomial

Polynomials

Polynomials

"Poly" means many

"nomials" means terms

So, polynomials means many terms

Definition of a Polynomial

An **algebraic expression** in which the variable(s) is / are raised to **non-negative integral exponents** is called a polynomial.

Standard Form of a Polynomial in x of Degree n

An algebraic expression of the form

$$\mathbf{p}(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where $a_0, a_1, a_2, \ldots, a_n$ are real numbers and $a_n \neq 0$,

is the standard form of a polynomial in x of degree n.

Terms Related to Polynomials

The Degree of a Polynomial $p(x)$ is the highest exponent to which x is raised.

The Value of a Polynomial $p(x)$ at $x = k$ is obtained by replacing $x = k$ in the polynomial expression.

A real number $'a'$ is a Zero of a Polynomial $p(x)$ if $p(a) = 0$.

Example

Degree = 2.

Value of $p(x)$ at $x = 1$ is $p(1) = 4(1)^2 - 1 = 3$.

$$p(x) = 4x^2 - 1$$

Zeroes of $p(x)$ are $\pm\frac{1}{2}$, since $p\left(\frac{1}{2}\right) = p\left(-\frac{1}{2}\right) = 0$.

Special Types of Polynomials

Based on Number of Terms

1 term → Monomial

Ex: $x, -5y$

2 terms → Binomial

Ex: $2x - 5$, $6y + 8$

3 terms → Trinomial

Ex: $x^2 - 3x + 2$

Based on Degree

Degree = 1 → Linear

Ex: $2y - 3$

Degree = 2 → Quadratic

Ex: $4x^2 + 5x - 2$

Degree = 3 → Cubic

Ex: $8x^3 - 5$

Relationship between Zeroes and Coefficients of a Polynomial

Quadratic Polynomial

General form: $p(x) = ax^2 + bx + c$

Sum of zeroes $= \alpha + \beta = \dfrac{-b}{a}$

Product of zeroes $= \alpha\beta = \dfrac{c}{a}$

Cubic Polynomial

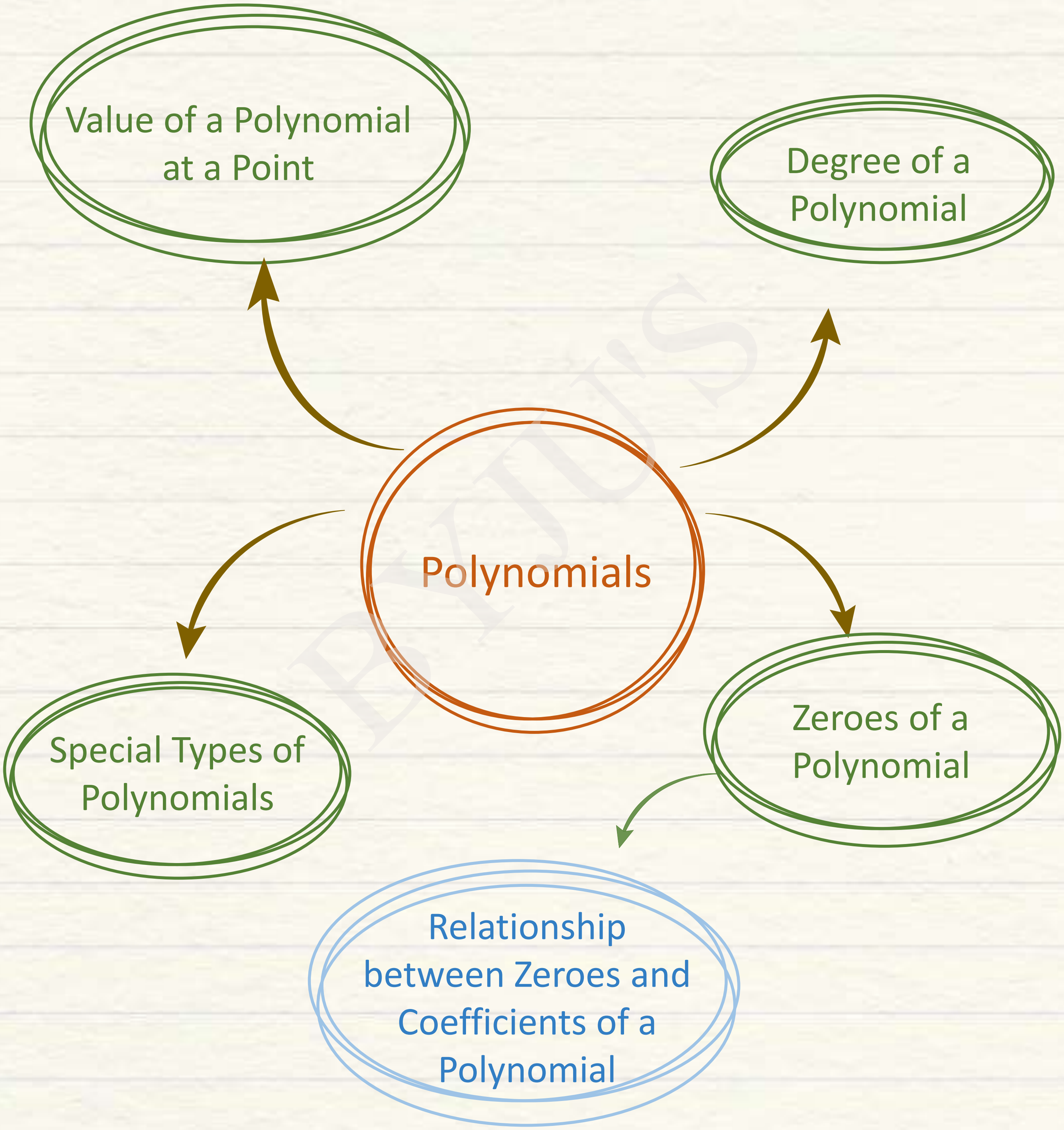
General form: $p(x) = ax^3 + bx^2 + cx + d$

Sum of zeroes $= \alpha + \beta + \gamma = \dfrac{-b}{a}$

Sum of product of zeroes taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a}$

Product of zeroes $= \alpha\beta\gamma = \dfrac{-d}{a}$

Mind Map


Value of a Polynomial at a Point
Degree of a Polynomial
Polynomials
Special Types of Polynomials
Zeroes of a Polynomial
Relationship between Zeroes and Coefficients of a Polynomial

MATHEMATICS

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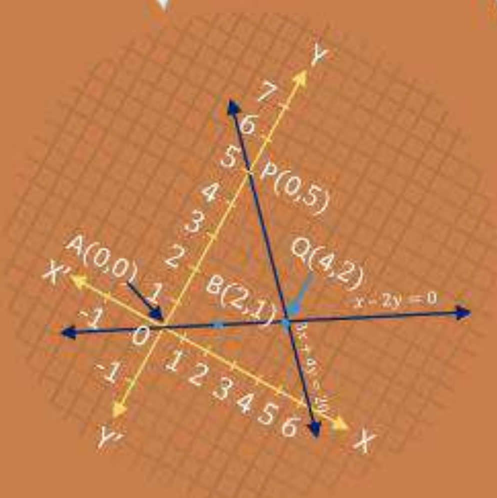
Pair of Linear Equations in Two Variables

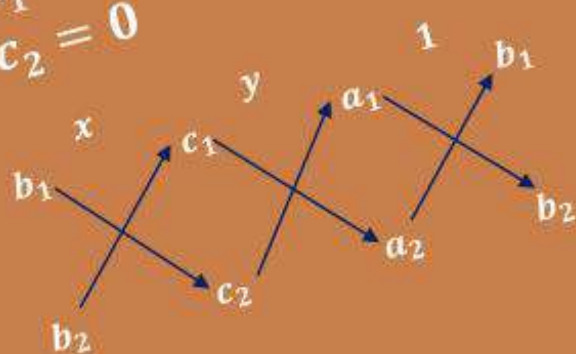
Topics

1. General Form of a Linear Equation
2. Types of Pairs of Linear Equations
3. Methods of Solving Pairs of Linear Equations



$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$



1. Linear Equations in Two Variables

General Form

Coefficients — Constant

$$ax + by + c = 0$$

Variables

where, **a and b** are **non-zero** real numbers

Pair of Linear Equations in Two Variables

Consider two different equations in x and y,

$$2x + 7y + 5 = 0$$

$$8x + 3y + 3 = 0$$

These two combined are known as pair of **linear equations** in two variables.

General Form of Pair of Linear Equations in Two Variables

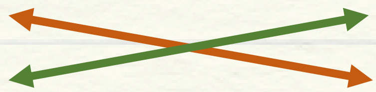
$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

2. Types of Pairs of Linear Equations

Parallel Lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Intersecting Lines



$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Coinciding Lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Parallel Lines → Inconsistent equations
No solution

Intersecting Lines and Coinciding Lines → Consistent equations
At least one solution

3. Methods of Solving Pairs of Linear Equations

- Methods of Solving
  - Graphical Method
  - Algebraic Methods
    - Substitution Method
    - Elimination Method
    - Cross-Multiplication Method

3.1 Graphical Method

$2x - 1y = -1$, $3x + 2y = 9$

Find points to construct lines on a graph paper for the two given equations

To construct a line, we need at least two point of the line, we find the value substituting values of x and y in the two equations.

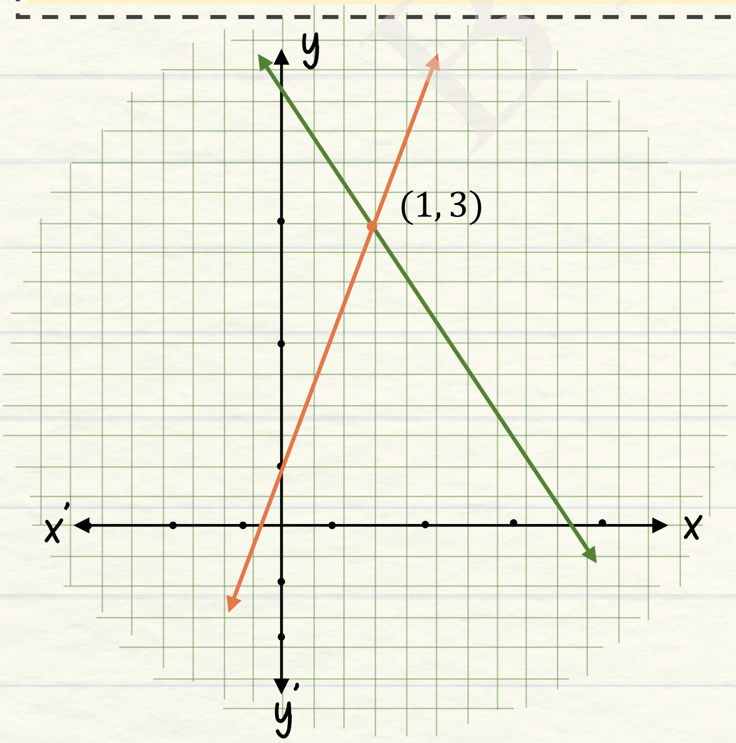
$2x - 1y = -1$

x	0	$-\frac{1}{2}$	1
y	1	0	3

$3x + 2y = 9$

x	0	3	1
y	$\frac{9}{2}$	0	3

Draw the two line on a graph and mark the points at which they intersect.



The x-coordinate and the y-coordinate of the point at which the two lines intersect is the solution(s) of the pair of equations.

3.2 Substitution Method

$x + y = 4$, $x - y = 2$

Take one of the equations and move 'y' to LHS and the rest to RHS to get the value of 'y' in terms of 'x'.

$$y = 4 - x$$

Substitute the obtained value of 'y' in the other equation to get the numerical value of 'x'.

$$x - y = 2$$
$$x - (4 - x) = 2$$
$$2x - 4 = 2$$
$$x = 3$$

Now, substitute the obtained value of 'x' in either of the equations to get the value of 'y'.

$$x + y = 4$$
$$3 + y = 4$$
$$y = 1$$

3.3 Elimination Method

$3x + 2y = 18$, $5x + 4y = 32$

1 Note down equations aligned to respective variables as shown.

$+3x$	$+2y$	$=$	$+18$
$+5x$	$+4y$	$=$	$+32$

2 Pick the variable which will be easier to eliminate.

$+3x$	$+2y$	$=$	$+18$
$+5x$	$+4y$	$=$	$+32$

3 Equalise the coefficients of the variable to be eliminated by multiplying every term of the equation with the same number.

$+3x$ $\times 2$	$+2y$ $\times 2$	$=$	$+18$ $\times 2$
$+5x$	$+4y$	$=$	$+32$

4 Subtract the second equation from the first equation by reversing all the signs.

$+6x$	$+4y$	$=$	$+36$
$-5x$	$-4y$	$=$	-32
$+x$	$+0y$	$=$	$+4$

5 Substitute the value of the now known variable into the simpler equation to get the value of the other variable.

We know that,

$x = 4$

And, $3x + 2y = 18$

$\Rightarrow 3 \times 4 + 2y = 18$

$\Rightarrow 12 + 2 = 18$

$\Rightarrow 2y = 6$

$\Rightarrow y = 3$

6 Verify the values obtained for x and y by putting them in the given equations

$3x + 2y = 18$,

LHS $= 3x + 2y$

$= 3 \times 4 + 2 \times 3$

$=$ RHS

$5x + 4y = 32$

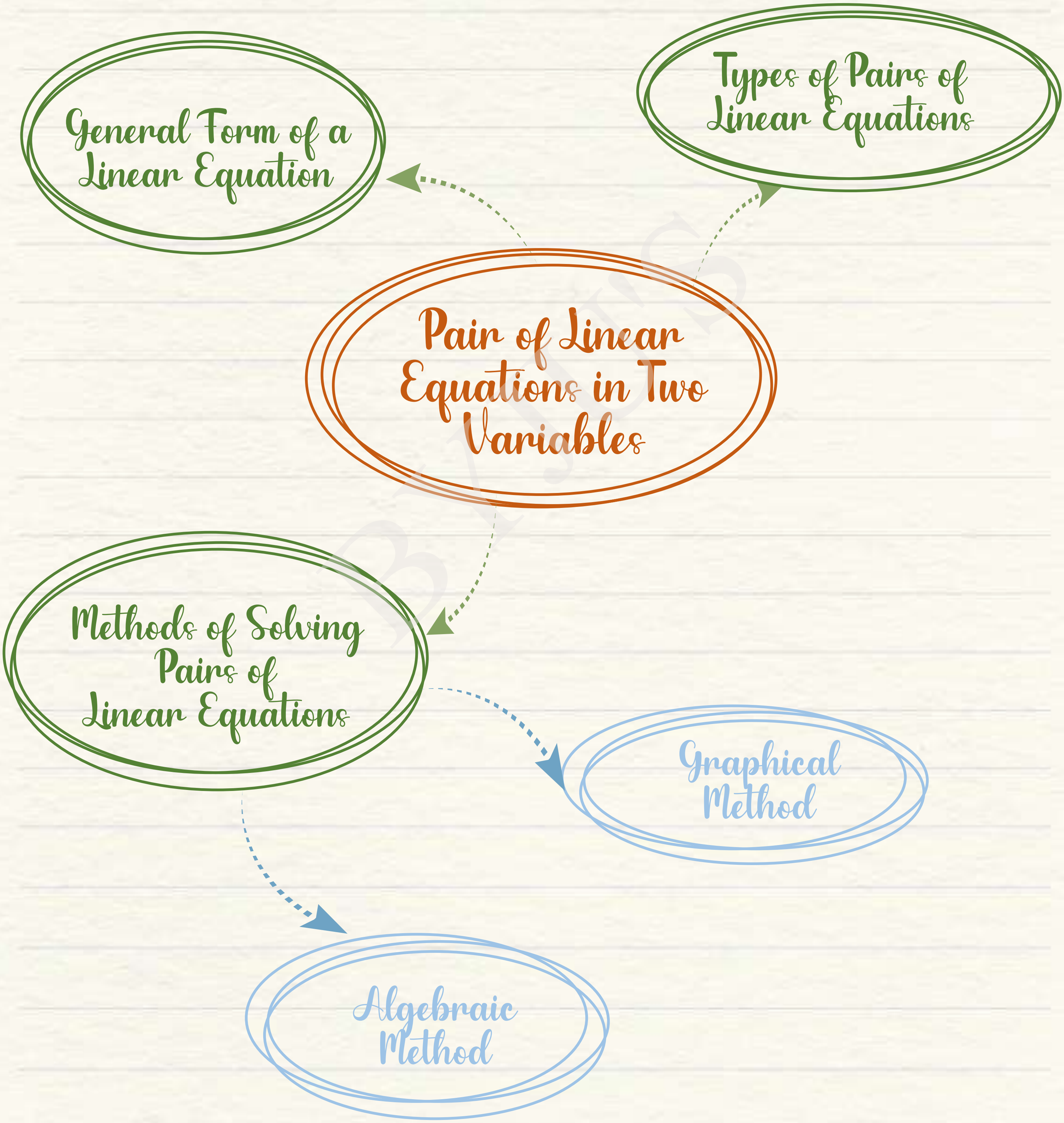
LHS $= 5x + 4y$

$= 5 \times 4 + 4 \times 3$

$=$ RHS

From the above, x = 4 and y = 3. Therefore, (4,3) is the solution of the simultaneous equations "3x + 2y = 18" and "5x + 4y = 32".

Mind Map



Pair of Linear Equations in Two Variables
General Form of a Linear Equation
Types of Pairs of Linear Equations
Methods of Solving Pairs of Linear Equations
Graphical Method
Algebraic Method

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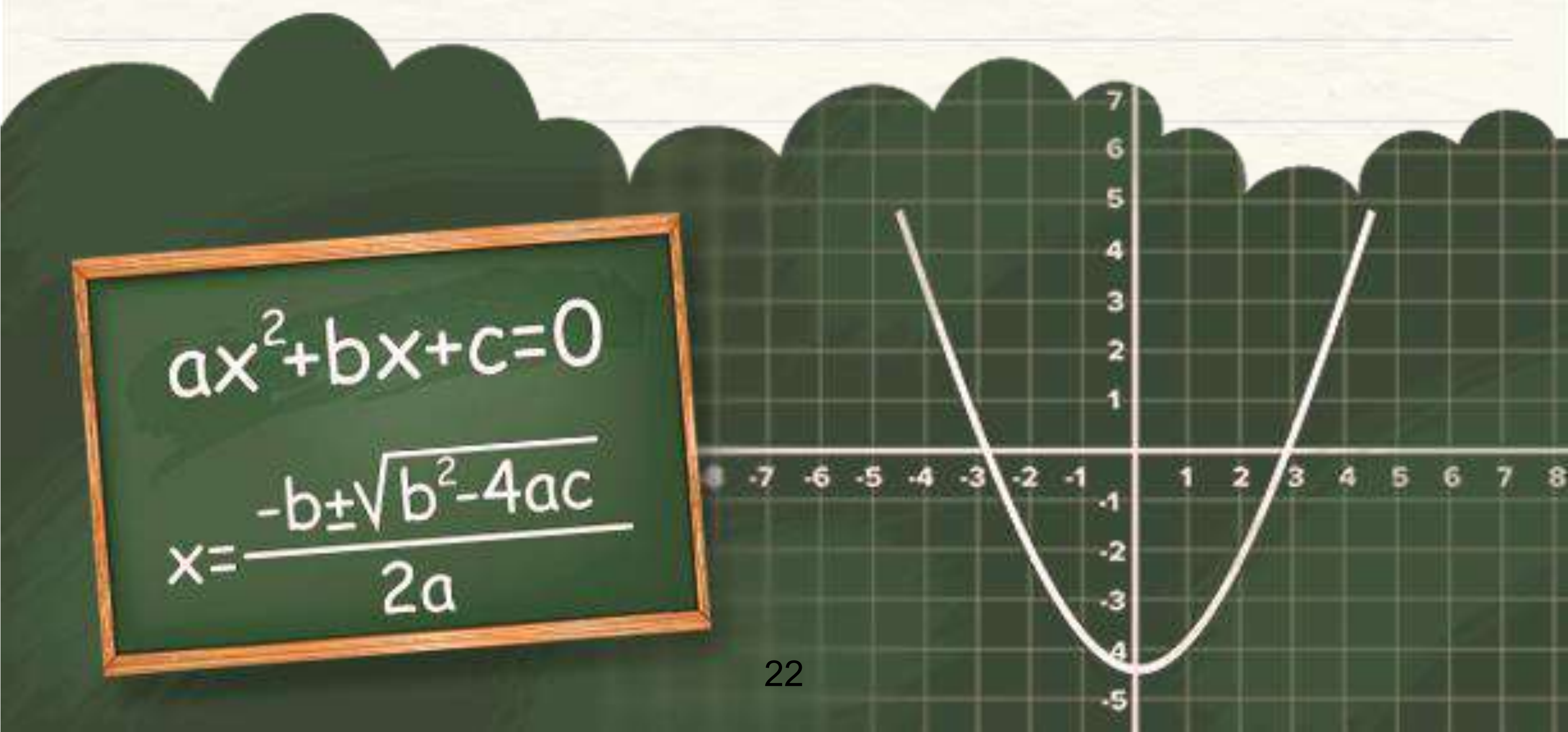
CHAPTER NOTES

Quadratic Equations

Topics

1. Standard Form of Quadratic Equations
2. Methods to Solve
3. Zeroes, Roots and Solutions
4. Nature of Roots



Standard Form

Degree

$$ax^{2} + bx + c = 0$$

Real numbers and $a \neq 0$

Important Terms

Zeroes

Zeroes are for quadratic **polynomial** P(x)

$P(x) = (x - 2)(x - 2)$

Zeroes, $x = 2$ & 2

Roots

Roots are for quadratic **equation**

$(x - 2)(x - 2) = 0$

Roots, $x = 2$ & 2

Solutions

Quadratic equation having **equal** and **identical** roots will have a **unique** solution.

$(x - 2)(x - 2) = 0$

$x = 2$ is the solution of the given equation

Methods to Solve Quadratic Equations

- Factorization
- Quadratic Formula

Factorization

General form:
$ax^2 + bx + c = 0.$

1. Split the middle term.

   Product of split terms = (a × c)

   $9x^2 - 3x - 2 = 0.$

   $9x^2 - 6x + 3x - 2 = 0.$

2. Factorize the equation

   $3x(3x - 2) + 1(3x - 2) = 0.$

   $(3x - 2)(3x + 1) = 0.$

3. Equate each factor to 0

   $(3x - 2)(3x + 1) = 0$

   $x = \frac{2}{3}$ or $x = -\frac{2}{3}$

Quadratic Formula

$$ax^2 + bx + c = 0$$

$$\text{Roots } (x) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i.e.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

where, $b^2 - 4ac \geq 0$

> Quadratic formula is used where factorization method is difficult to apply.

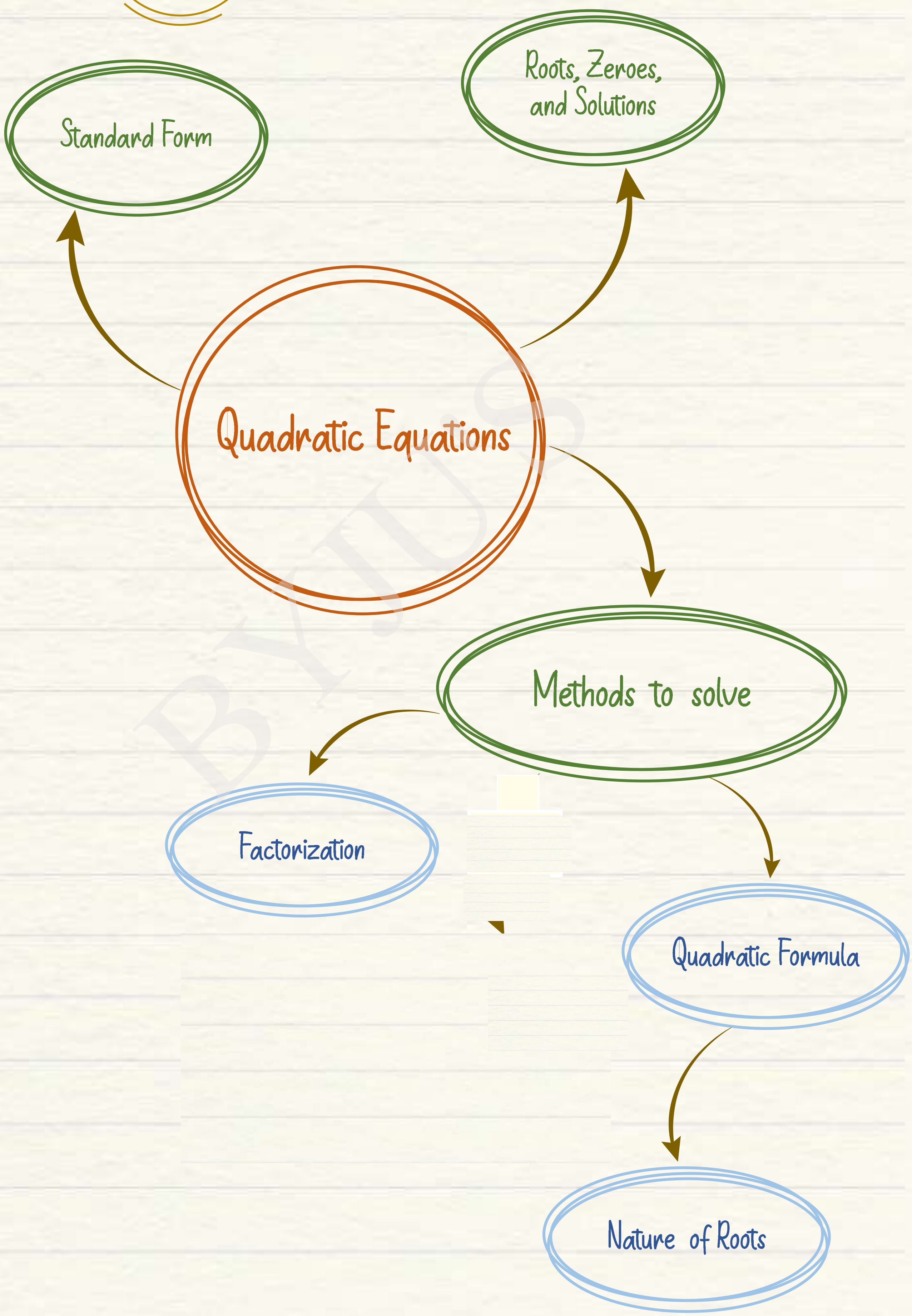
Nature of Roots

Discriminant (D) = "$b^2 - 4ac$".

Nature of Roots

	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Type of Roots	Real & Distinct Roots	Real & Equal Roots	No Real Roots
Value of Roots	$\frac{-b - \sqrt{D}}{2a}$, $\frac{-b + \sqrt{D}}{2a}$	$\frac{-b}{2a}$, $\frac{-b}{2a}$	Not Valid

Mind Map



Quadratic Equations
Standard Form
Roots, Zeroes, and Solutions
Methods to solve
Factorization
Quadratic Formula
Nature of Roots

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CHAPTER NOTES

Arithmetic Progressions

Topics

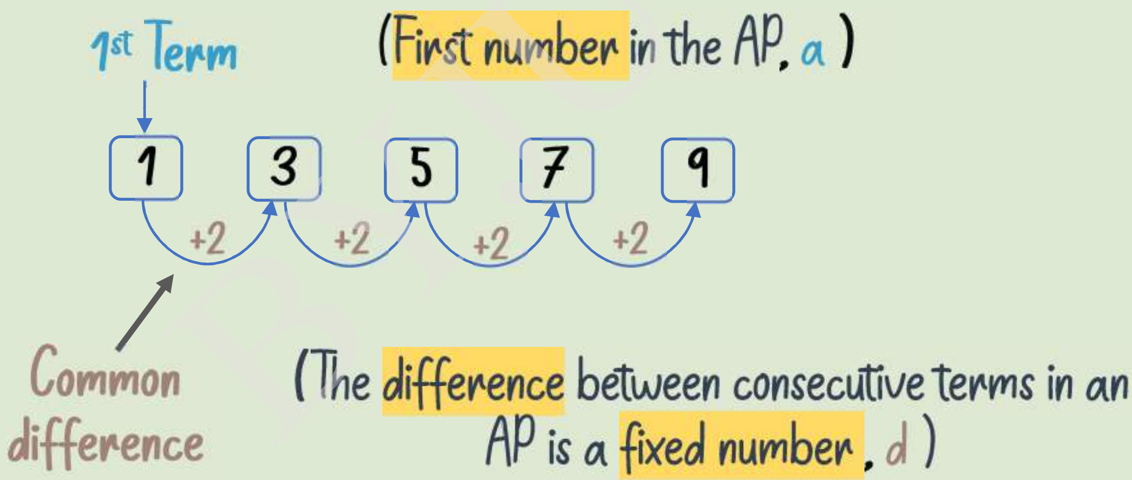
1. Arithmetic progression
2. Types of an Arithmetic Progression
3. General form of an AP
4. n^{th} Term of an AP
5. Sum of first n terms of an AP
6. Arithmetic mean

1. Arithmetic Progressions

Definition

An arithmetic progression is a sequence of numbers in which each term is obtained by adding a fixed number to the preceding term, except the first term.

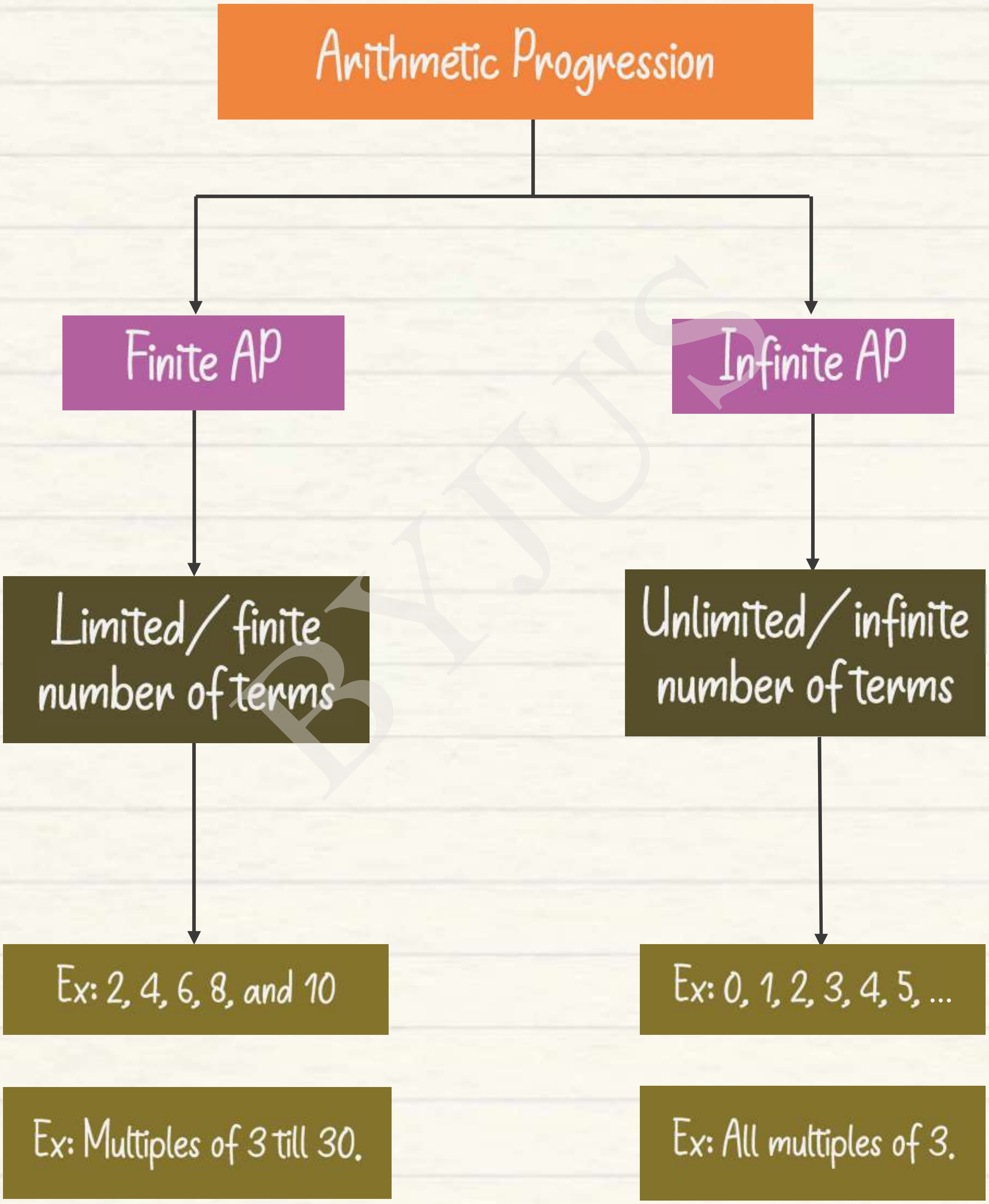
Example



The common difference can be

- Negative → Decreasing AP
- Zero → Constant AP
- Positive → Increasing AP

2.Types of an Arithmetic Progression



3. General Form of an AP

A sequence of the form

$a, a+d, a+2d, a+3d, a+4d$ and so on,

where a is the first term and d is the common difference.

4. n^{th} Term of an AP

$$a_n = \{a + (n-1)d\}$$

where a is the first term,

d is the common difference

n is the number of terms in the sequence and

a_n is the nth term.

5. Sum of First n Terms in an AP

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

(When first term (a) and common difference (d) are known)

$$S_n = \frac{n}{2}(a + l)$$

(When first term (a) and last term (l) are known)

where n is the number of terms in the sequence and S_n is the sum of first n terms

6. Arithmetic Mean

$$b = \frac{a + c}{2}$$

If a, b and c are in AP, then,

b is the arithmetic mean of a and c.

Important Formulae

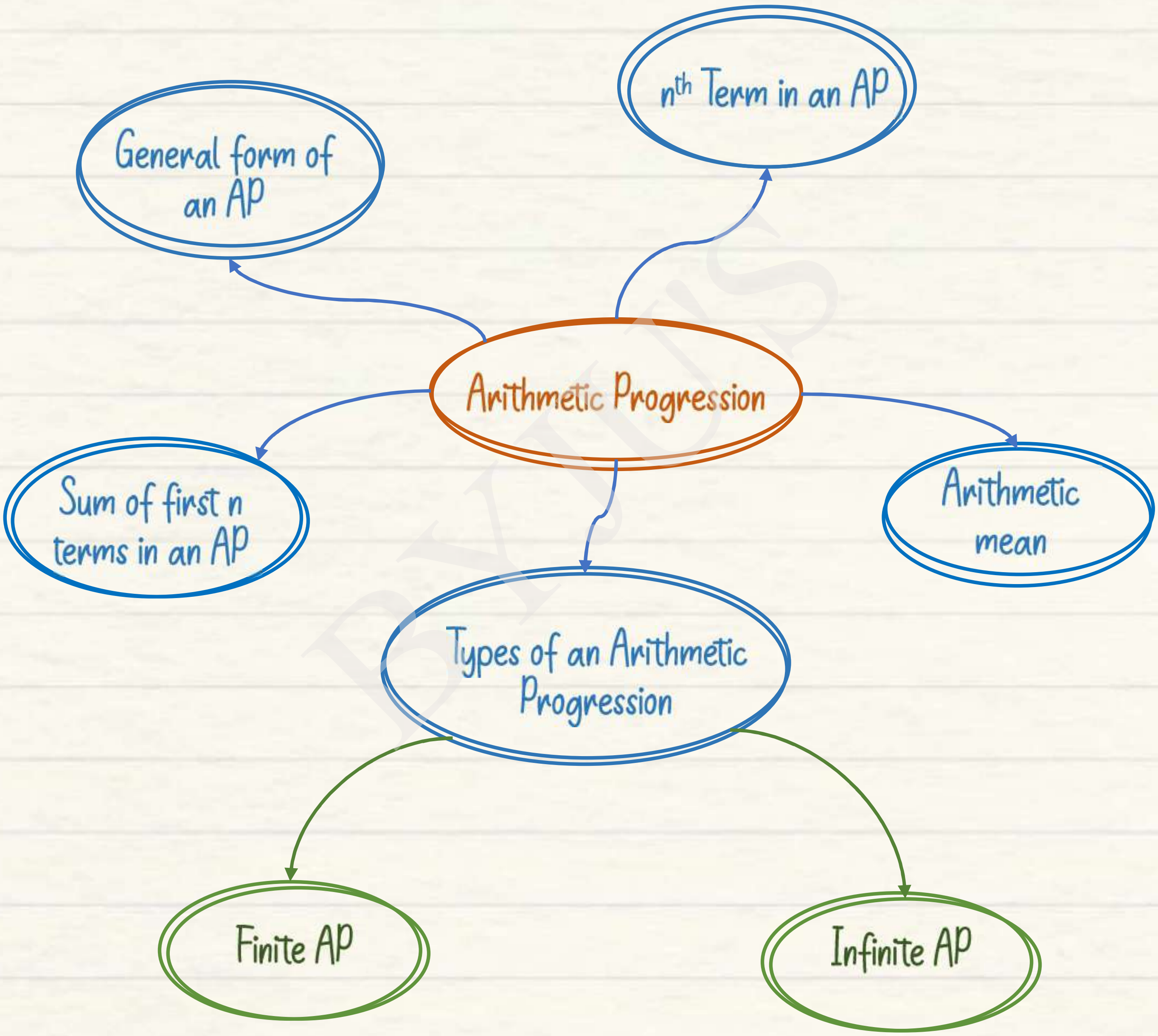
n^{th} Term of an AP	$a_n = a + (n-1)d$
Sum of first n terms in an AP (Where first term (a) and common difference (d) are known)	$S_n = \frac{n}{2}\{2a + (n-1)d\}$
Sum of first n terms in an AP (Where first term (a) and last term (l) are known)	$S_n = \frac{n}{2}(a + l)$
Arithmetic Mean (b) (a, b and c are in AP)	$b = \frac{a + c}{2}$

Tips/Points to be Remembered

While solving questions containing consecutive terms, following assumptions can be made to simplify:

NUMBER OF TERMS	CONSECUTIVE TERMS	FIRST TERM	COMMON DIFFERENCE
3	$(a-d), a, (a+d)$	$(a-d)$	d
4	$(a-3d), (a-d), (a+d), (a+3d)$	$(a-3d)$	$2d$
5	$(a-2d), (a-d), a, (a+d), (a+3d)$	$(a-2d)$	d

Mind Map


General form of an AP
n^{th} Term in an AP
Arithmetic Progression
Sum of first n terms in an AP
Arithmetic mean
Types of an Arithmetic Progression
Finite AP
Infinite AP

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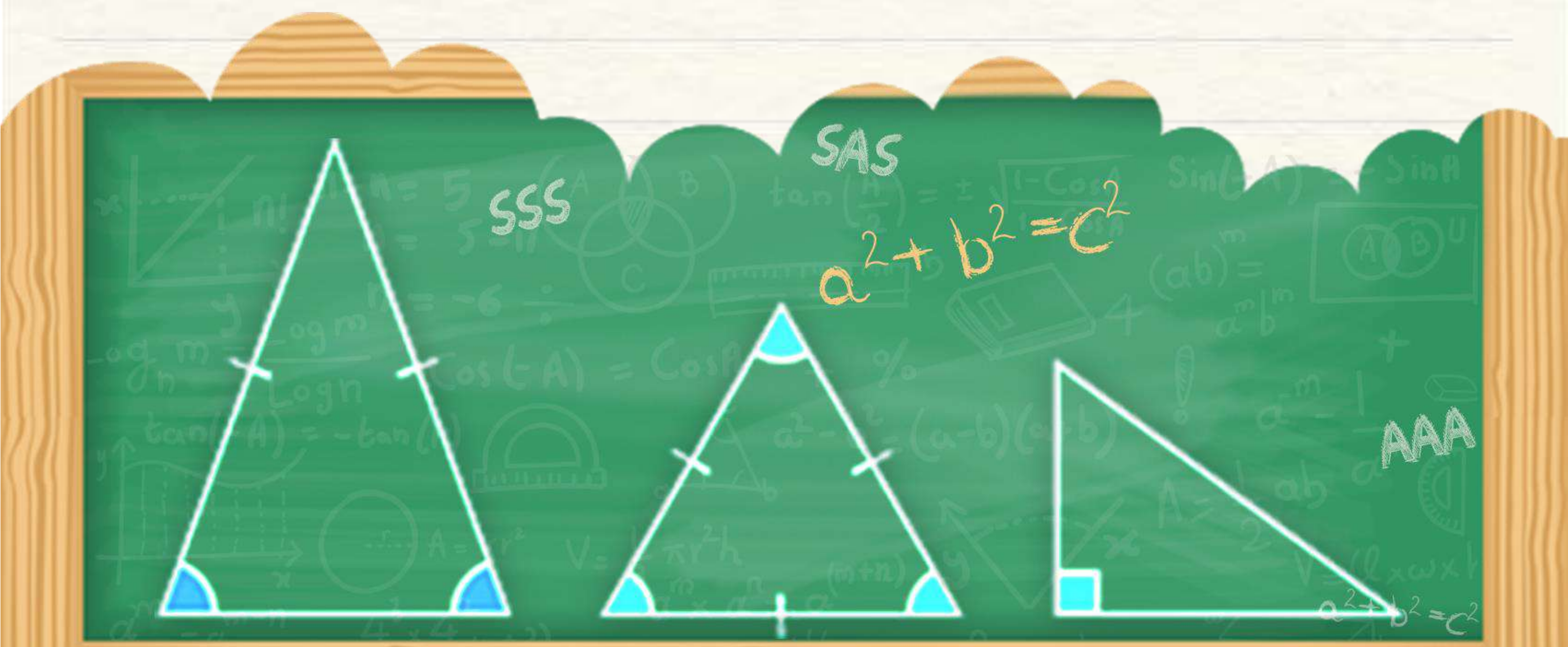
CHAPTER NOTES

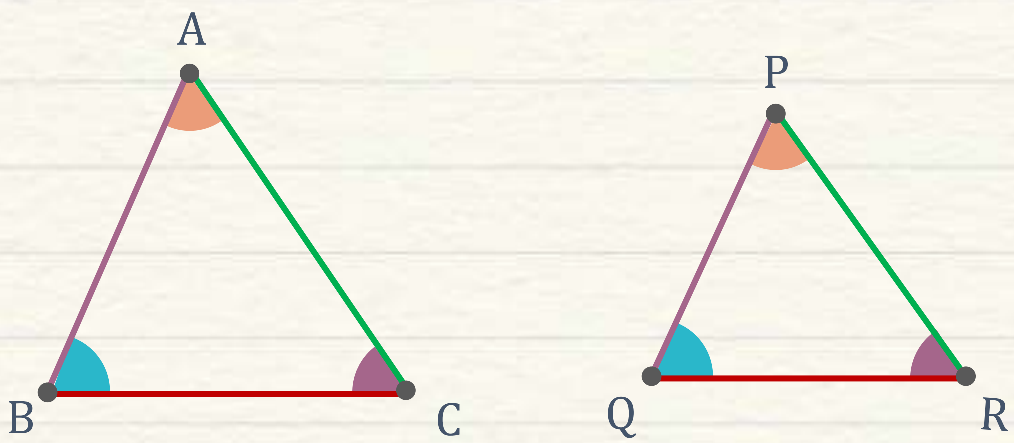
Triangles

Topics

1. Similar Triangles
2. Criteria of Similarity of Triangles
3. Pythagoras Theorem
4. Basic Proportionality Theorem



Similar Triangles



Similar Triangles → Same Shape

Similar Triangles → Different or Same size

Relation between Corresponding Sides and Angles

- Two triangles are similar, if
  - Their corresponding angles are equal.

    $\angle A = \angle P$

    $\angle B = \angle Q$

    $\angle C = \angle R$

  - Their corresponding sides are in the same ratio.

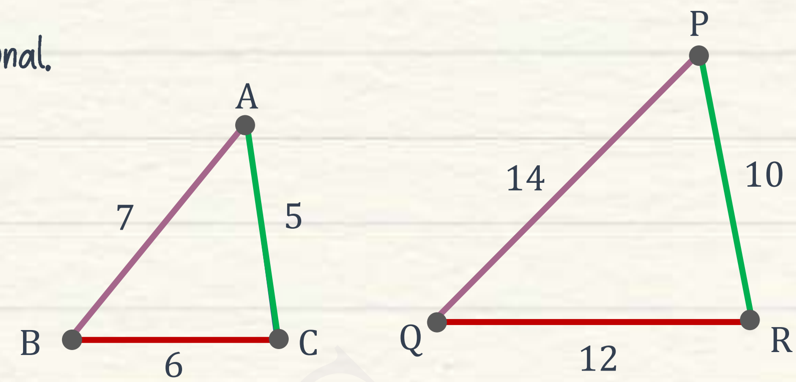
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = k$$

Criteria for Similarity of Triangles

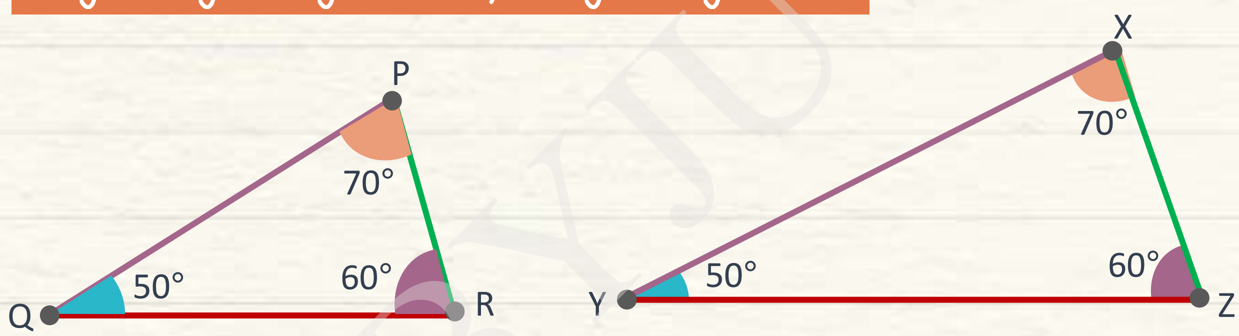
Side-Side-Side (SSS)

★ Corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$



Angle-Angle-Angle (AAA) / Angle-Angle (AA)



★ Corresponding angles are equal.

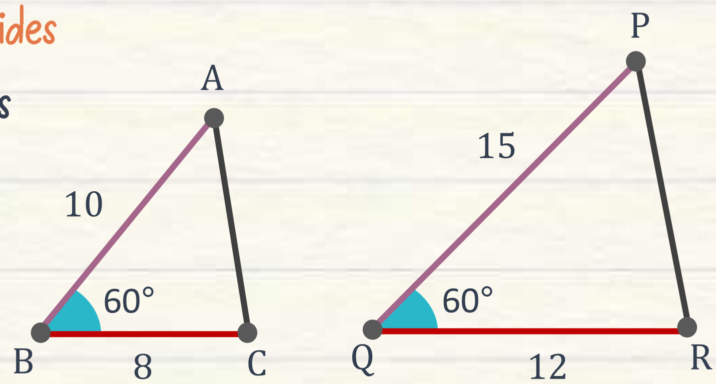
★ Triangles are similar even if a pair of corresponding angles are equal.

Side-Angle-Side (SAS)

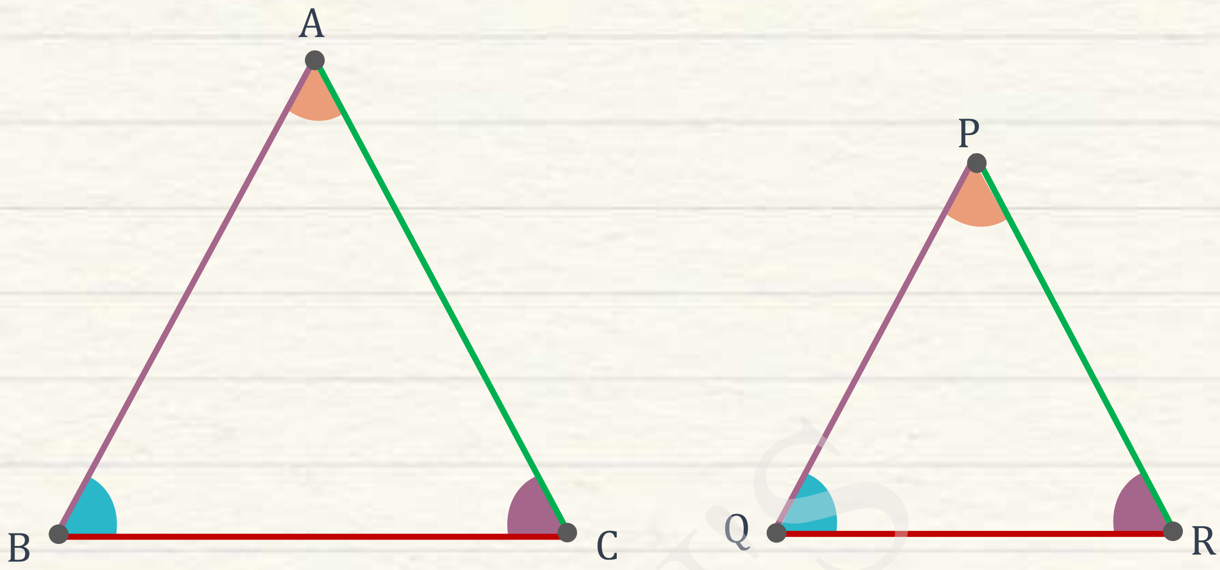
★ Pair of adjacent corresponding sides are proportional and one angle is equal.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2}{3}$$

$\angle B = \angle Q$



Ratio of Areas of Similar Triangles



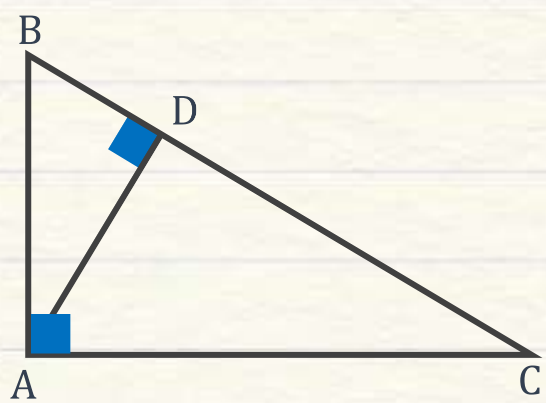
Ratio of Area of Similar Triangles

$$\frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Properties of Right-Angled Triangles

Similarity of triangles when a perpendicular is drawn from the vertex of the right angle.

$\Delta ABC \sim \Delta ADC \sim \Delta ADB$ (AA Similarity)



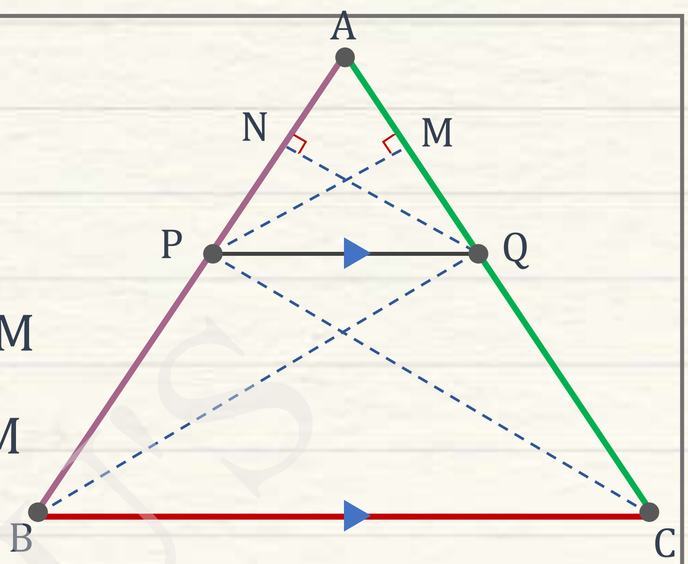
All the three triangles have:

- A right-angle.
- A common angle.

Basic Proportionality Theorem

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Proof:



Area of $\Delta APQ = \frac{1}{2} \times AP \times QN$

Area of $\Delta PBQ = \frac{1}{2} \times PB \times QN$

Area of $\Delta APQ = \frac{1}{2} \times AQ \times PM$

Area of $\Delta QCP = \frac{1}{2} \times QC \times PM$

Now,

$$\frac{\text{Area of } \Delta\ APQ}{\text{Area of } \Delta\ PBQ} = \frac{\frac{1}{2} \times AP \times QN}{\frac{1}{2} \times PB \times QN} = \frac{AP}{PB} \quad \text{............(1)}$$

Similarly,

$$\frac{\text{Area of } \Delta\ APQ}{\text{Area of } \Delta\ QCP} = \frac{\frac{1}{2} \times AQ \times PM}{\frac{1}{2} \times QC \times PM} = \frac{AQ}{QC} \quad \text{..........(2)}$$

The triangles drawn between the same parallel lines and on the same base have equal areas.

$\therefore$ Area of ΔPBQ = Area of ΔQCP ....(3)

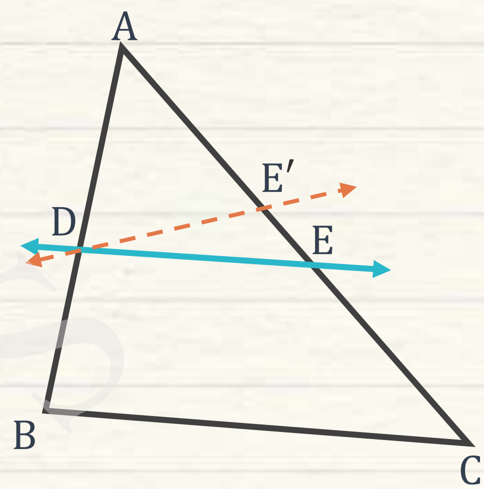
From (1), (2) and (3)

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Converse of Basic Proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Proof:



If $\frac{AD}{DB} = \frac{AE}{EC}$, then DE || BC.

Suppose a line DE, intersects the two sides of a triangle AB and AC at D and E, such that:

$$\frac{AD}{DB} = \frac{AE}{EC} \quad ......(1)$$

Assume DE is not parallel to BC. Now, draw a line DE' parallel to BC.

Hence, by Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE'}{E'C} \quad ......(2)$$

From eq. 1 and 2, we get

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

Adding 1 on both the sides

$$\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1 \Rightarrow \frac{AE+EC}{EC} = \frac{AE'+E'C}{E'C}$$

$$\frac{AC}{EC} = \frac{AC}{E'C} \Rightarrow \text{So, } EC = E'C$$

This is possible only when E and E' coincides.

But DE' || BC

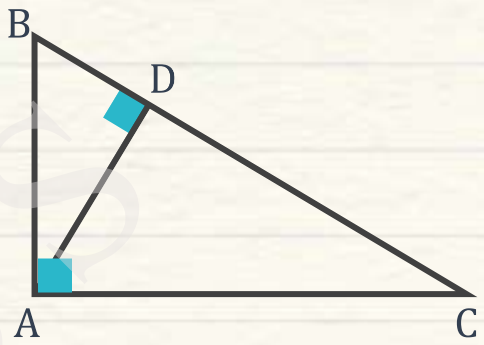
∴ DE || BC.

Properties of Right-Angled Triangles

Pythagoras Theorem

In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides.

Proof:



$\triangle ADB \sim \triangle ABC$

$\therefore \dfrac{AD}{AB} = \dfrac{AB}{AC}$ (corresponding sides of similar triangles)

$AB^2 = AD \times AC$ ....(1)

Also, $\triangle ADC \sim \triangle ABC$

$\therefore \dfrac{CD}{BC} = \dfrac{BC}{AC}$ (corresponding sides of similar triangles)

$BC^2 = CD \times AC$ ....(2)

(1) + (2)

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$AB^2 + BC^2 = AC\,(AD + CD)$

Since, $AD + CD = AC$

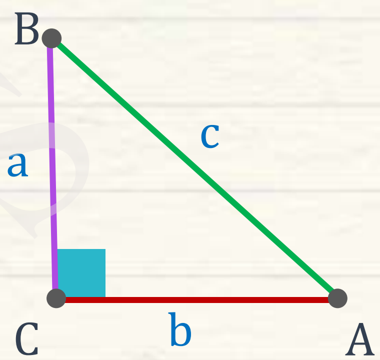
$\therefore AC^2 = AB^2 + BC^2$

Converse of Pythagoras Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

Proof:

Construct another triangle, $\triangle EGF$, such as $AC = EG$ and $BC = FG$.



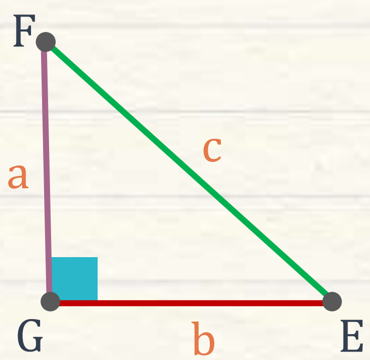
In $\triangle EGF$, by Pythagoras Theorem:

$EF^2 = EG^2 + FG^2 = b^2 + a^2$ ......(1)

In $\triangle ABC$, by Pythagoras Theorem:

$AB^2 = AC^2 + BC^2 = b^2 + a^2$ ......(2)

From (1) and (2)



$EF^2 = AB^2$

$EF = AB$

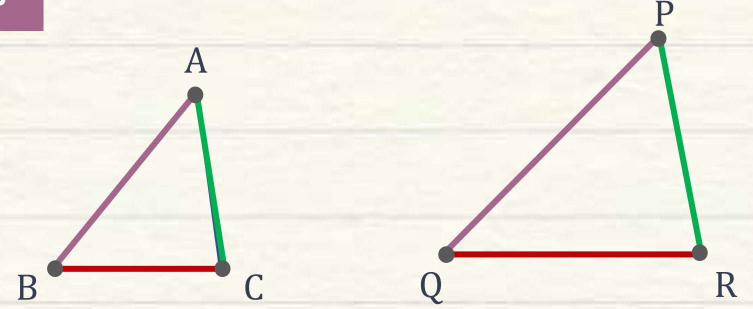
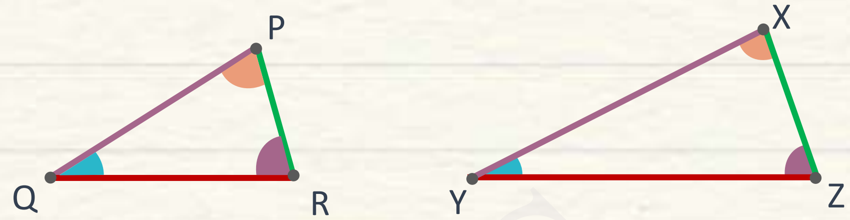
$\Rightarrow \triangle ACB \cong \triangle EGF$ (By SSS)

$\Rightarrow \angle C$ is right angle

$\therefore$ $\triangle ABC$ is a right triangle.

Important Theorems and Formulae

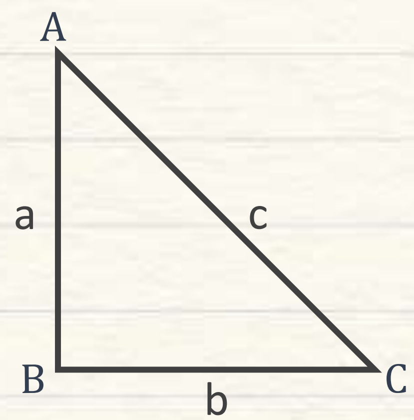
Similarity of Triangles

SSS	
AAA/AA	
SAS	

Pythagoras Theorem

- In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides.

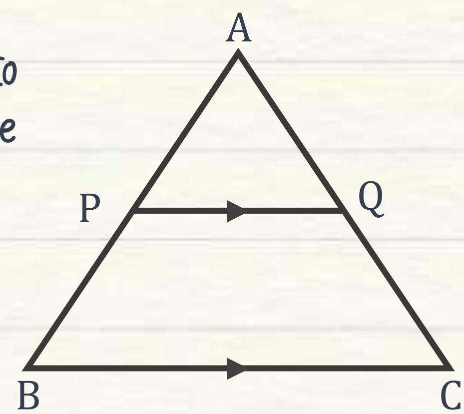
$$a^2 + b^2 = c^2$$



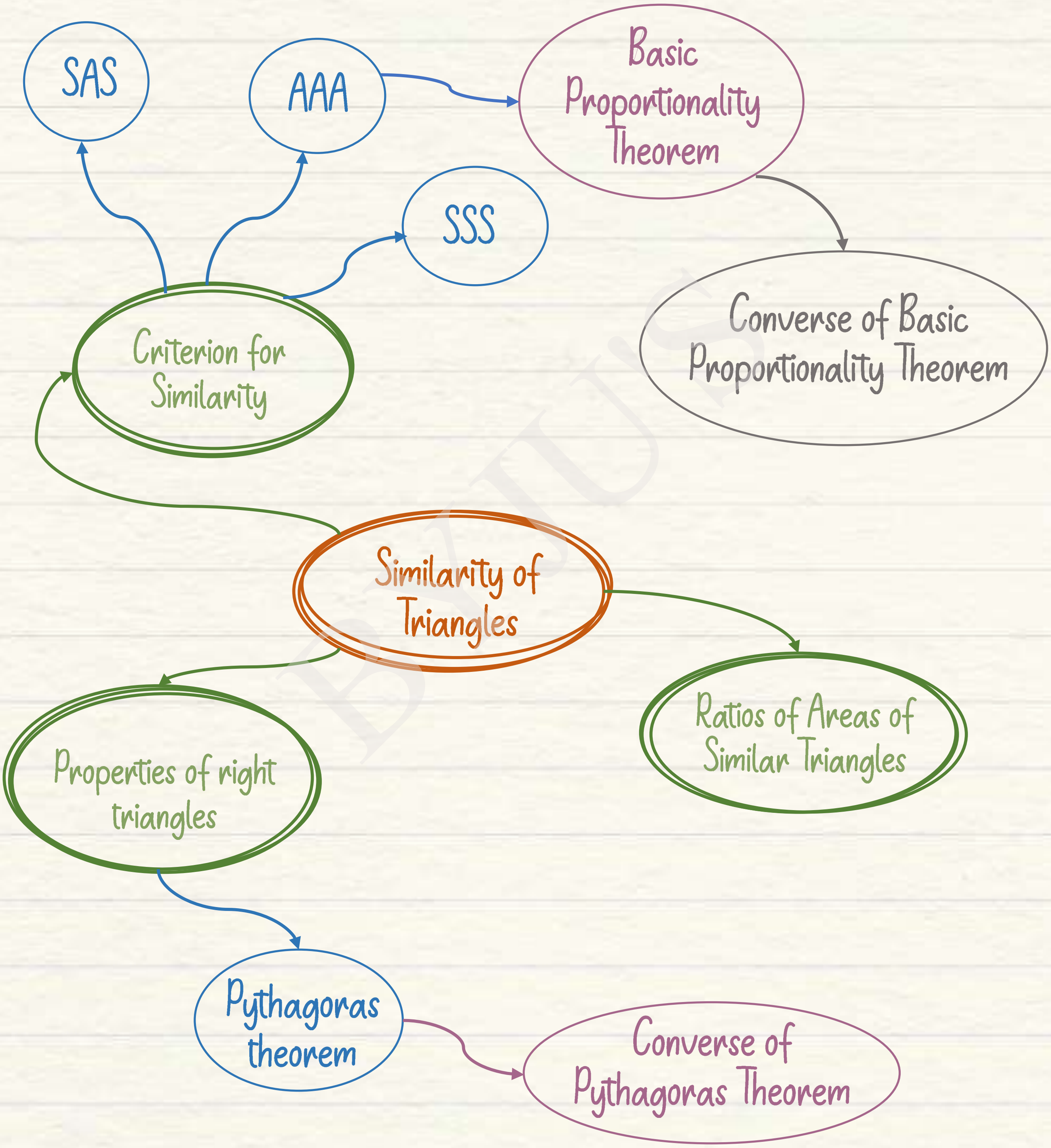
Basic Proportionality Theorem

- If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$PQ \parallel BC, \quad \frac{AP}{PB} = \frac{AQ}{QC}$$



Mind Map


SAS
AAA
Basic Proportionality Theorem
SSS
Criterion for Similarity
Converse of Basic Proportionality Theorem
Similarity of Triangles
Ratios of Areas of Similar Triangles
Properties of right triangles
Pythagoras theorem
Converse of Pythagoras Theorem

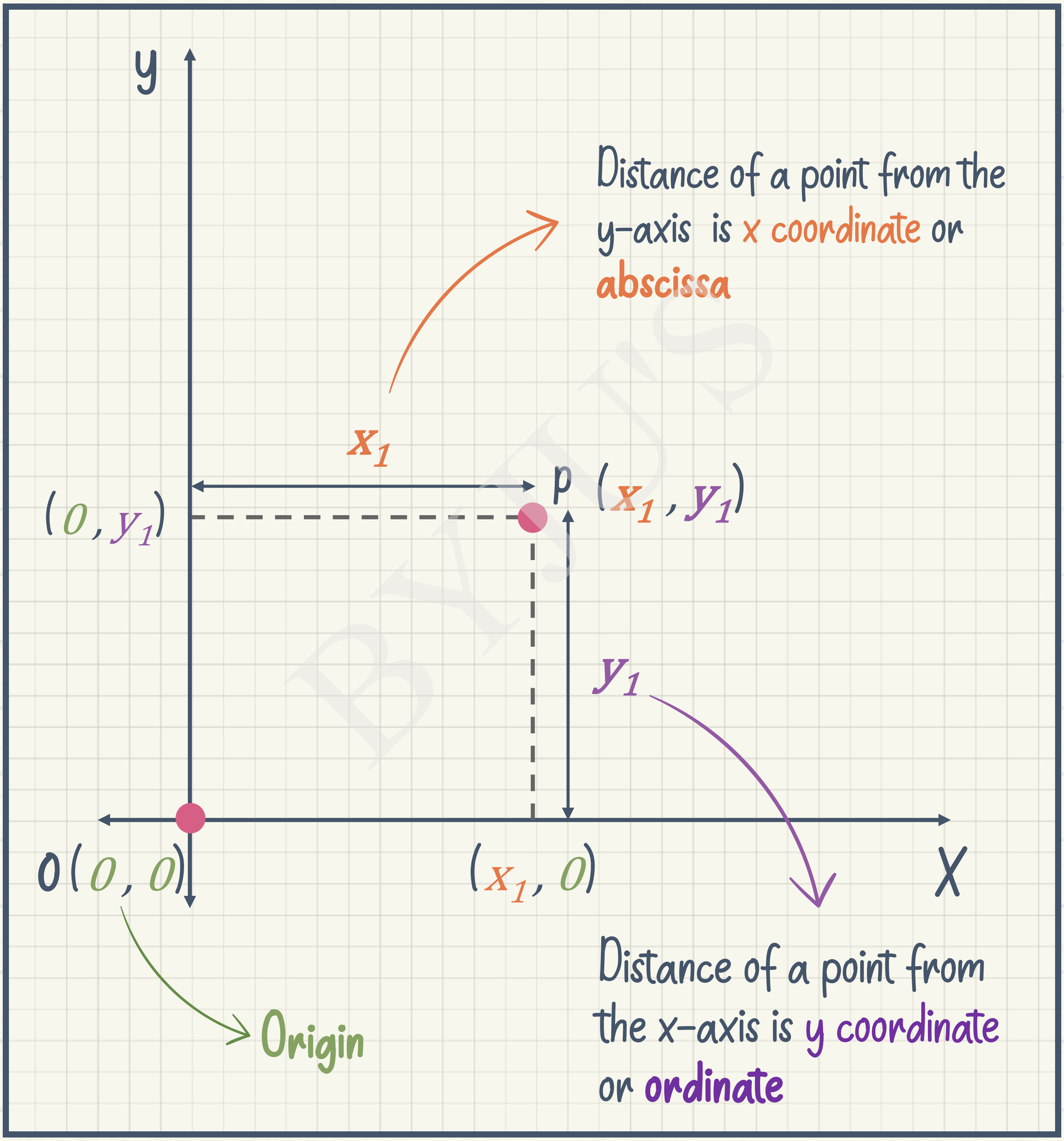
BYJU'S CHAPTER NOTES

Coordinate Geometry

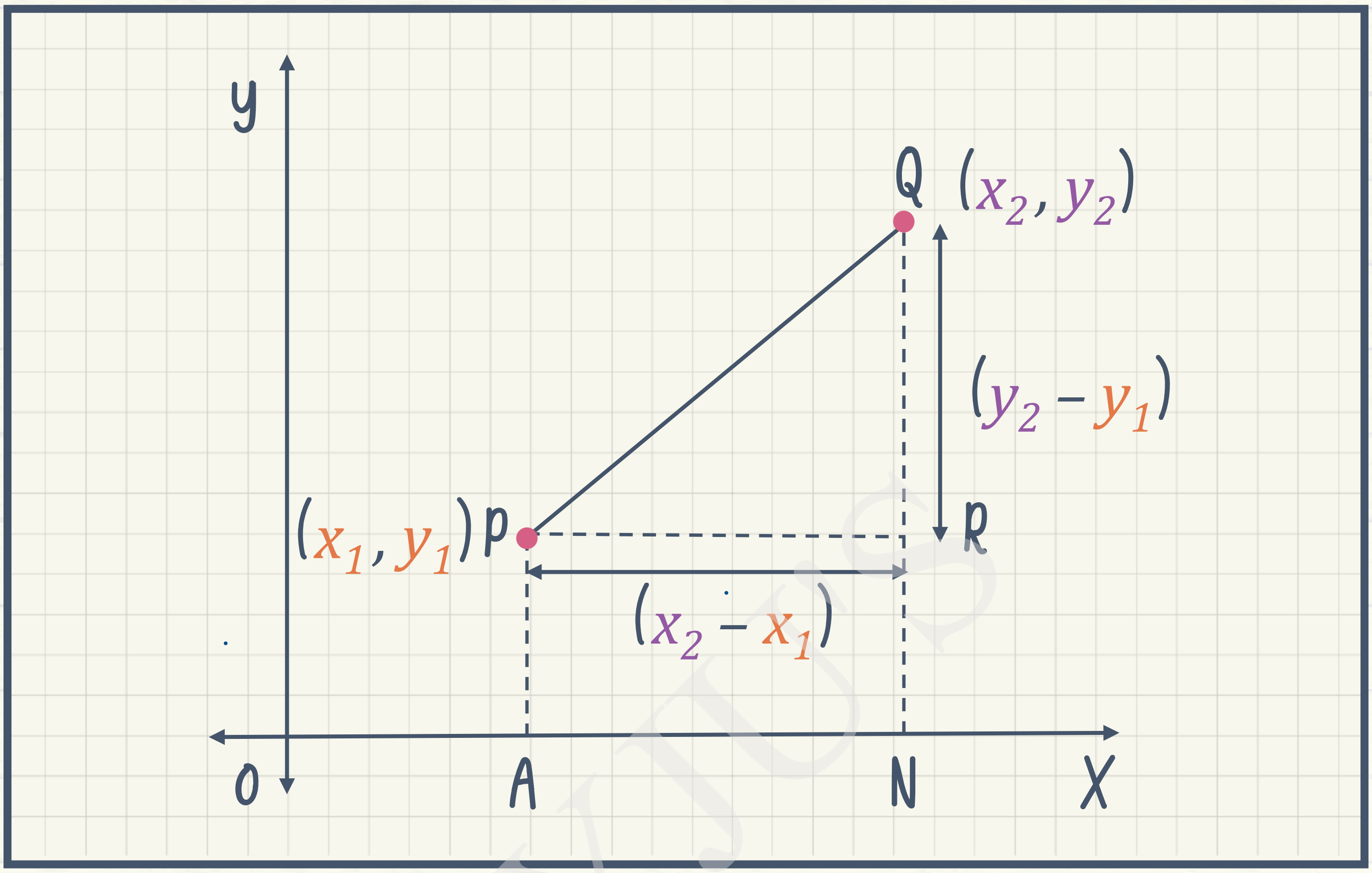
Topics

1. Fundamentals
2. Distance Formula
3. Section Formula
   3.1 Mid-Point Formula

Fundamentals



Distance Formula



Steps to Derive

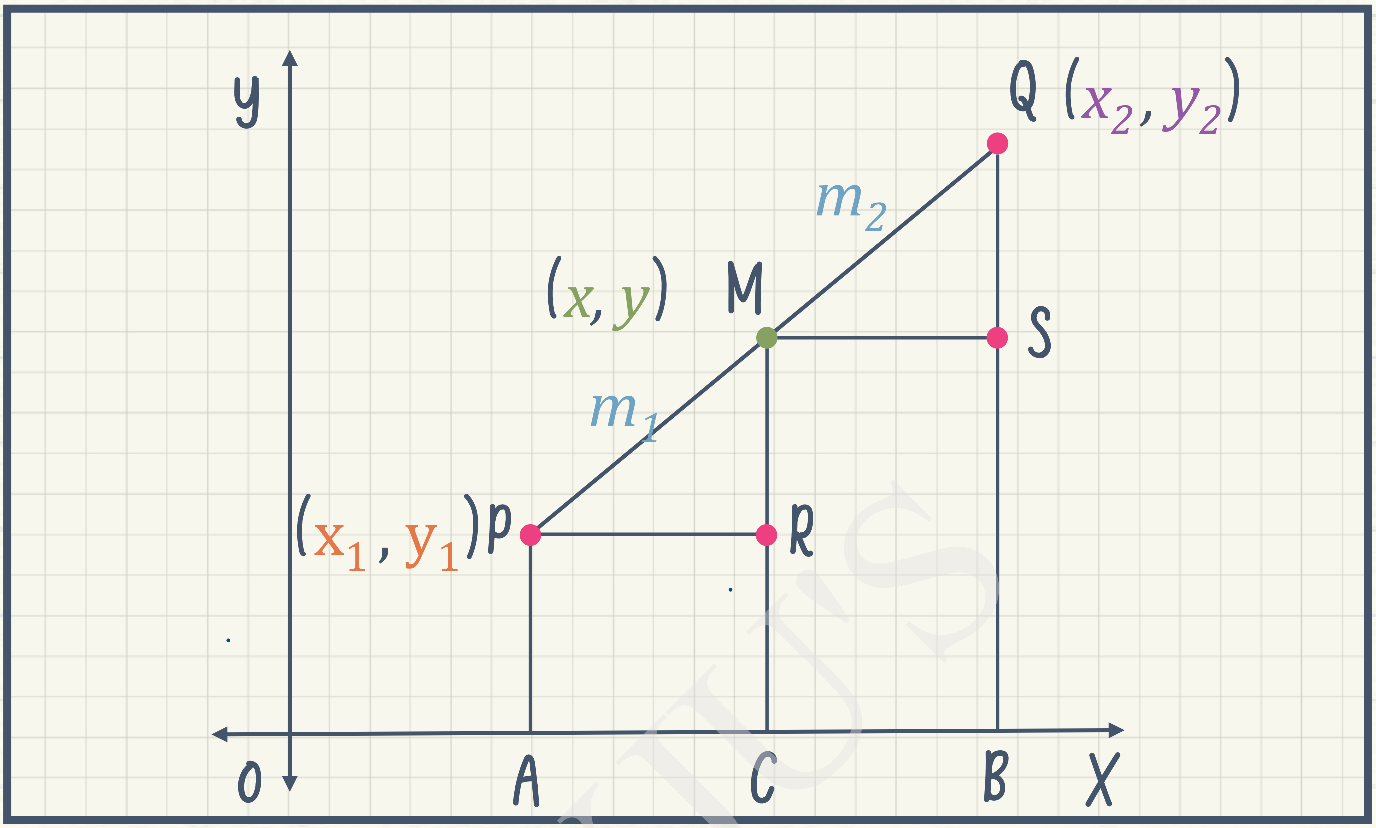
Using Pythagoras theorem:

$$PQ = \sqrt{(PR)^2 + (QR)^2}$$

Now, PR = $(x_2 - x_1)$ and QR = $(y_2 - y_1)$

Distance, PQ $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Section Formula



Steps to Derive

△PRM ~ △ MSQ (Similar triangles)

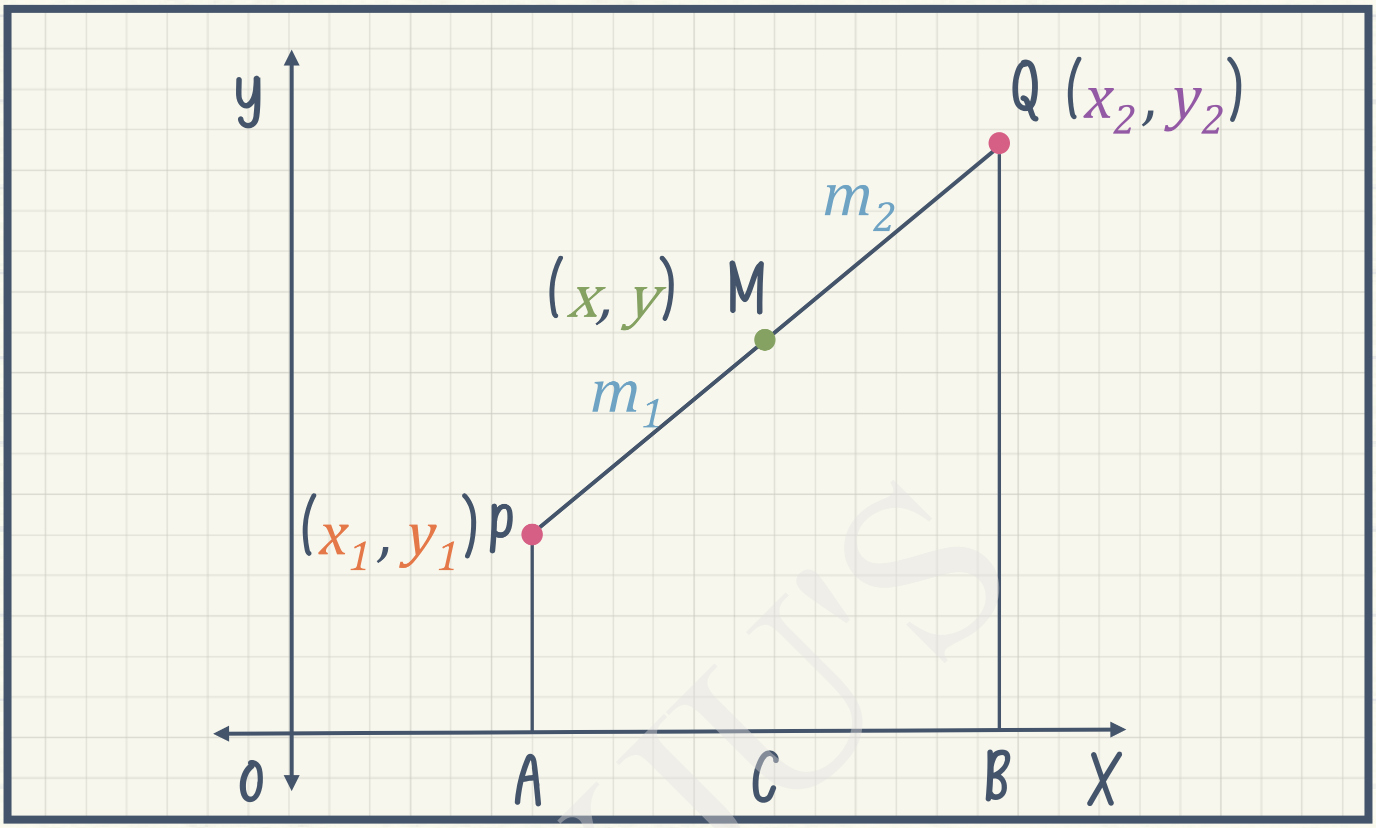
$$\frac{PM}{MQ} = \frac{PR}{MS} = \frac{RM}{SQ}$$

$$\frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$$

On solving for x and y separately:

$$M(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Mid-Point Formula



Steps to Derive

Section Formula

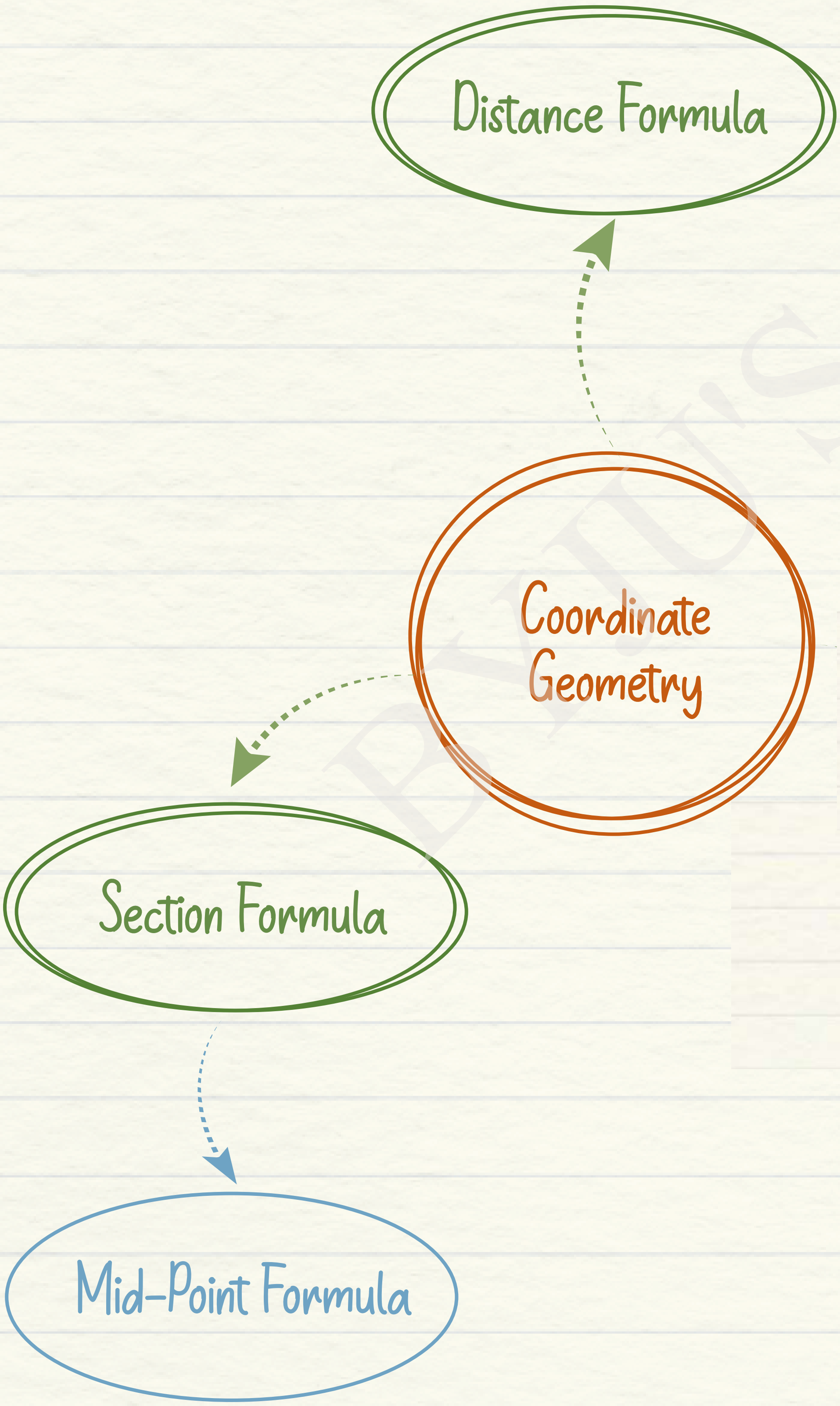
$$M(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

M is the mid point, so $m_1 : m_2 = 1 : 1$

$\therefore$ $m_1 = 1$ and $m_2 = 1$

$$\mathbf{M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Mind Map

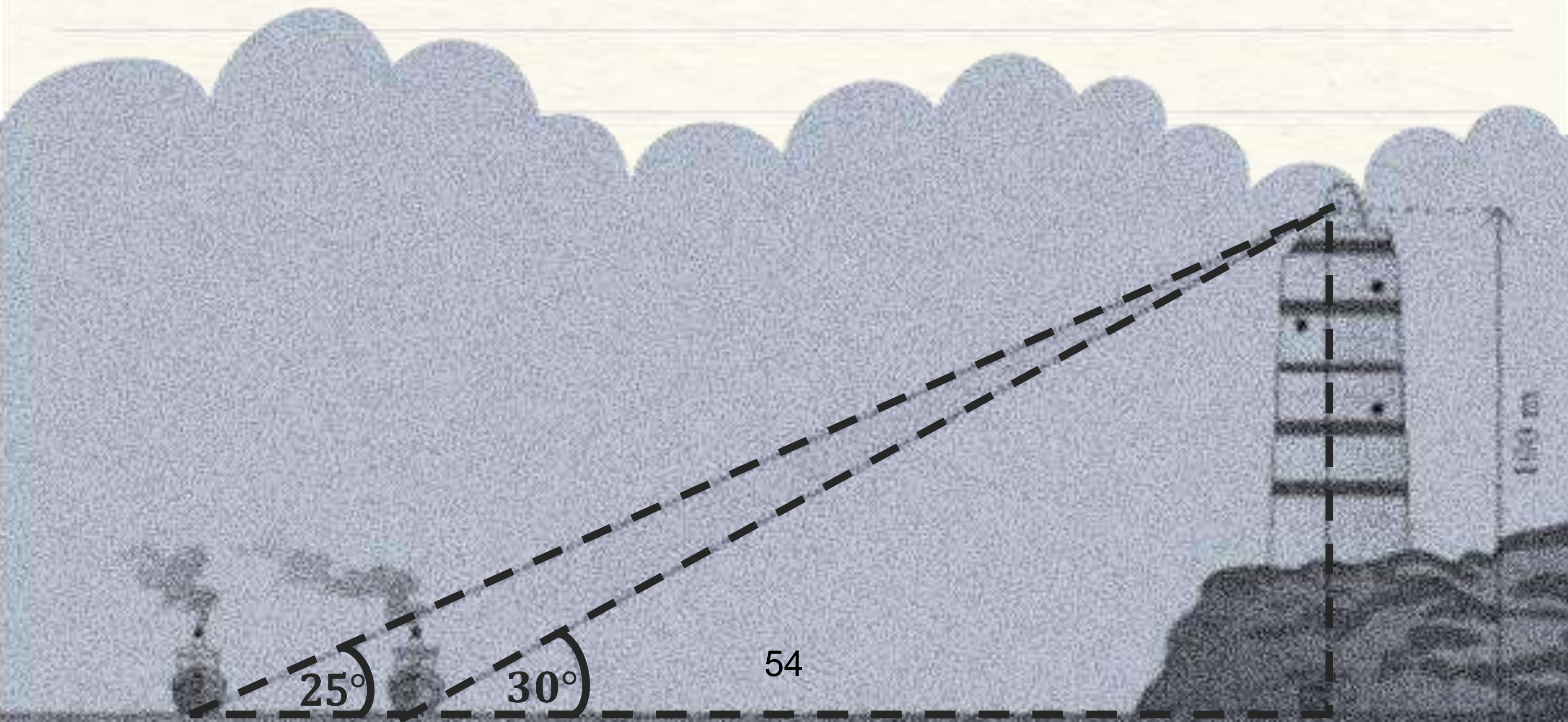

Distance Formula
Coordinate Geometry
Section Formula
Mid-Point Formula

BYJU'S CHAPTER NOTES

Introduction to Trigonometry

Topics

1. Trigonometric Ratios
2. Trigonometric Ratios of standard angles
3. Trigonometric Identities

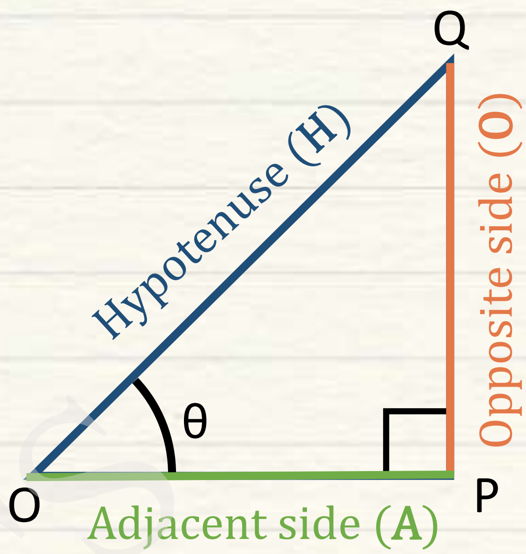


Trigonometric Ratios

SOH: $\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

CAH: $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\text{Opposite}}{\text{Adjacent}}$



MULTIPLICATIVE INVERSE

i. $\sin\theta$ → i. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{Hypotenuse}}{\text{Opposite}}$

ii. $\cos\theta$ → ii. $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{Hypotenuse}}{\text{Adjacent}}$

iii. $\tan\theta$ → iii. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\text{Adjacent}}{\text{Opposite}}$

Trigonometric Ratios of Standard Angles

- With just the values of sin θ, we can calculate all other trigonometric ratios for standard angles.

An idea to learn the sin values

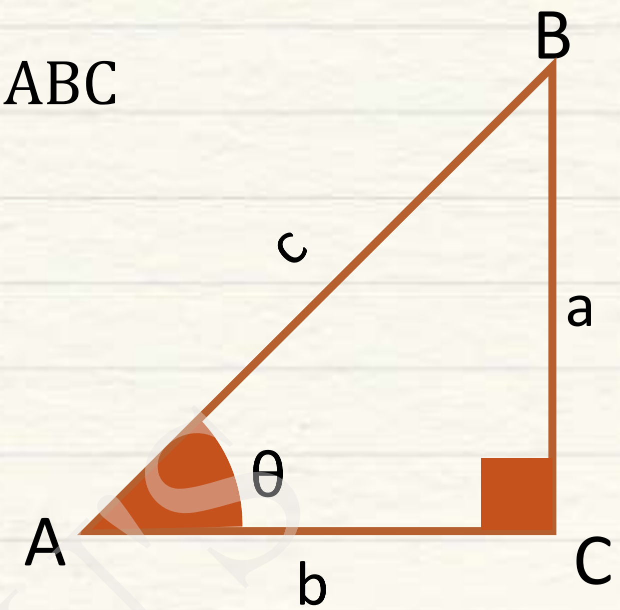
$\boldsymbol{\theta}$	0°	30°	45°	60°	90°
1. Write numbers from 0 to 4 in order.	0	1	2	3	4
2. Divide every number by 4	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
3. Take the square root of every number	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
4. Simplify	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
sin $\boldsymbol{\theta}$	sin 0°	sin 30°	sin 45°	sin 60°	sin 90°

Trigonometric Ratios of Standard Angles

Ratios \ Angles	Logic	0°	30°	45°	60°	90°
sinθ	sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	Reverse sinθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	$\frac{\sin\theta}{\cos\theta}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecθ	$\frac{1}{\sin\theta}$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secθ	$\frac{1}{\cos\theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotθ	$\frac{1}{\tan\theta}$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Proof of Trigonometric Identities

In a right - Angled Triangle Δ ABC



In Δ ABC we know that

$\sin\theta = \frac{a}{c}$................1

$\cos\theta = \frac{b}{c}$................2

By Pythagoras Theorem,

$a^2 + b^2 = c^2$

Dividing both sides by c^2

$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$

$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$

From 1 and 2,

$\sin^2\boldsymbol{\theta} + \cos^2\boldsymbol{\theta} = \mathbf{1}$

Proof of $1 + \tan^2\theta = \sec^2\theta$

We know that

$\sin^2\theta + \cos^2\theta = 1$

Dividing both the sides by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

⇒ $\tan^2\theta + 1 = \sec^2\theta$

Proof of $1 + \cot^2\theta = \csc^2\theta$

We know that

$\sin^2\theta + \cos^2\theta = 1$

Dividing both the sides by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

⇒ $1 + \cot^2\theta = \csc^2\theta$

Three Basic Trigonometric Identities

1

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

2

$\sec^2\theta - \tan^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

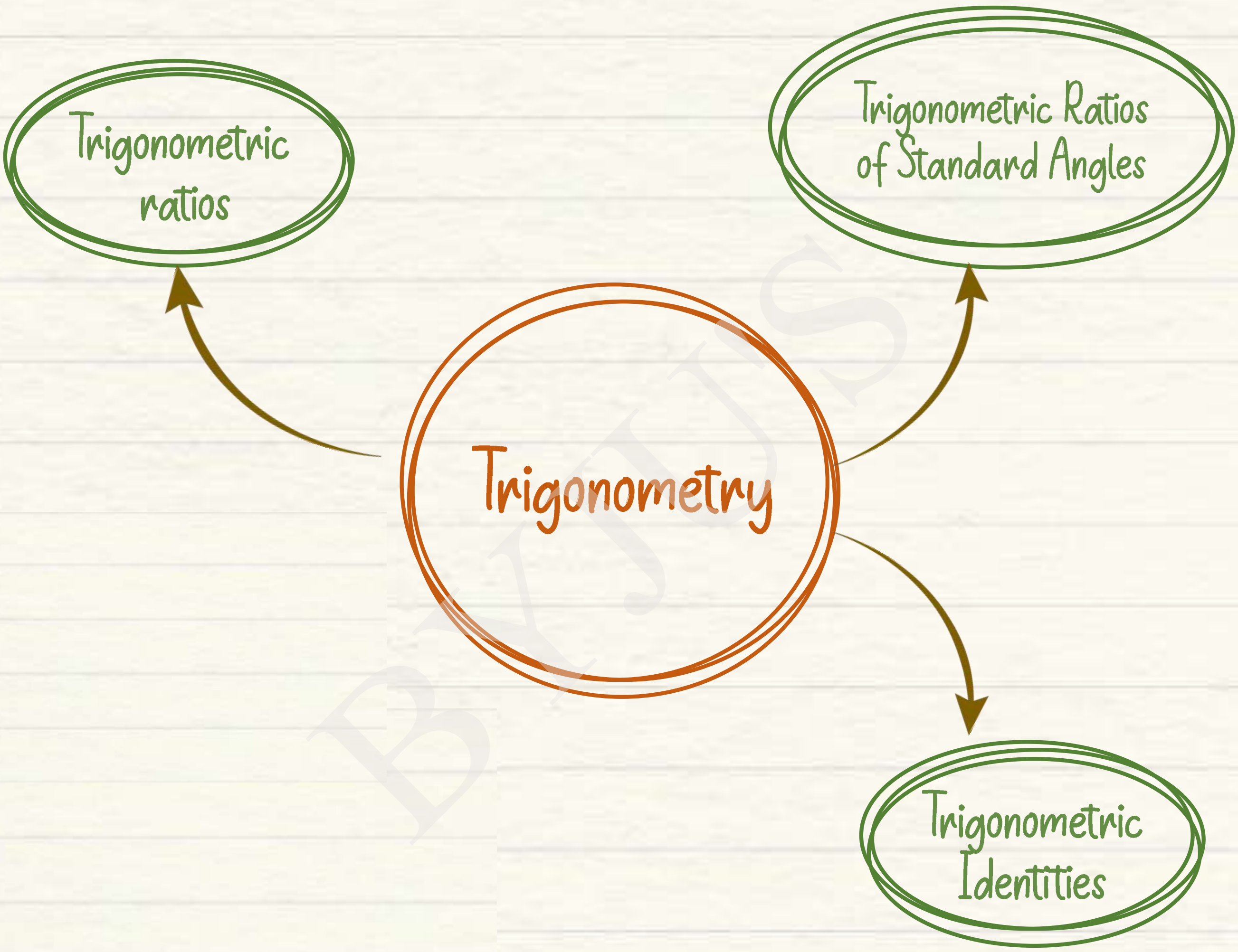
3

$\csc^2\theta - \cot^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$\csc^2\theta - 1 = \cot^2\theta$

Mind Map


Trigonometric ratios
Trigonometric Ratios of Standard Angles
Trigonometry
Trigonometric Identities

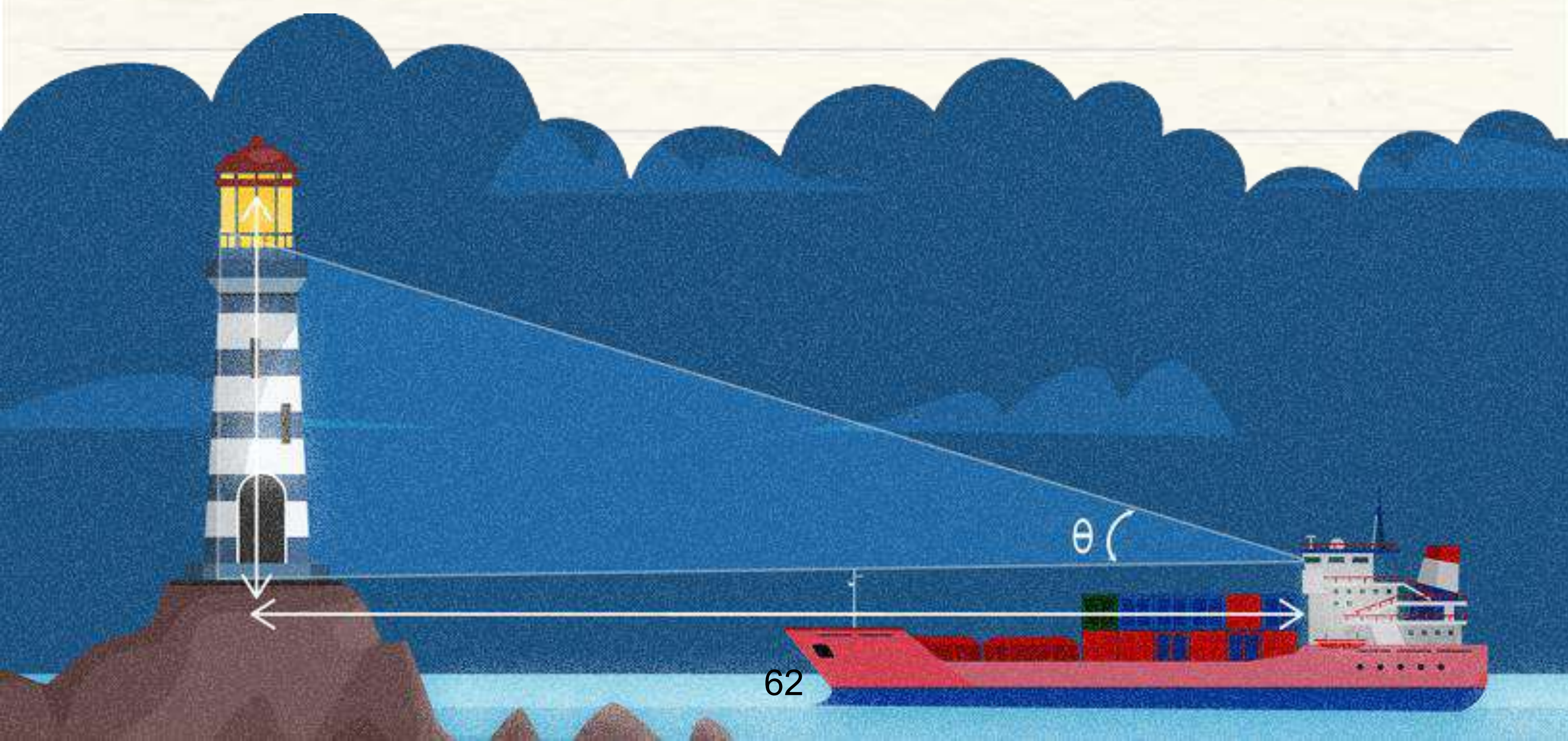
MATHEMATICS

BYJU'S
CHAPTER NOTES

Some Applications of Trigonometry

Topics

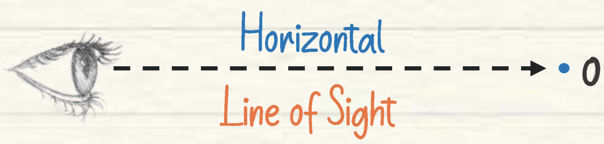
1. Basic Terminologies
2. Assumptions made while solving
3. Trigonometric Ratios of Some Common Angles
4. Method of Solving Questions



1. Basic Terminologies

Line of Sight

The line drawn from the eyes of an observer to a point on the object viewed.

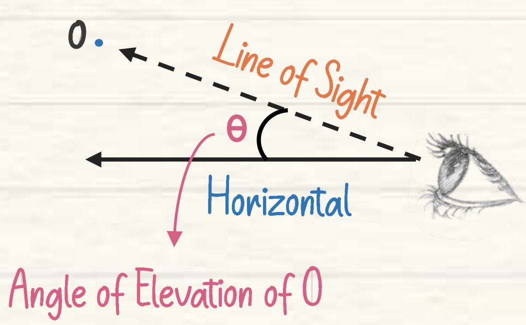


If the object to be viewed is straight ahead, then the line of sight is the same as the horizontal level.

Angle of Elevation

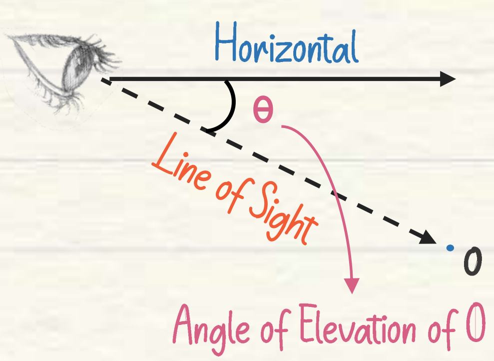
Look Above

The angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.



Angle of Depression

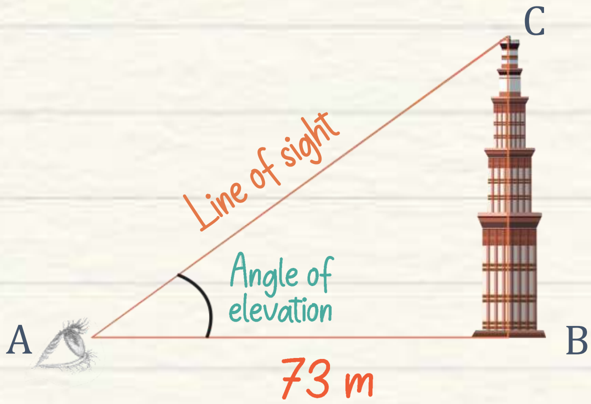
Look Below



The angle formed by the line of sight with the horizontal when the point being viewed is below the horizontal level.

2. Assumptions Made While Solving

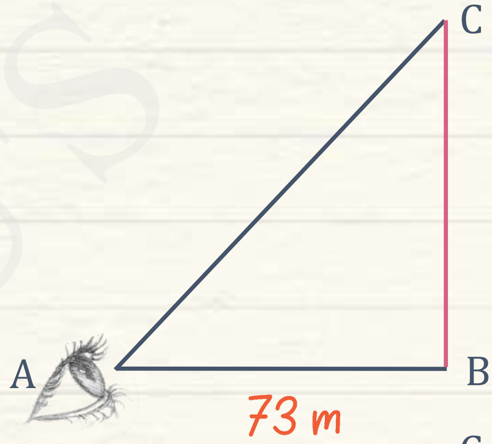
The angle of elevation of the top of the Qutub Minar, 73 m away from its base is 45°.



Steps to Draw the figure:

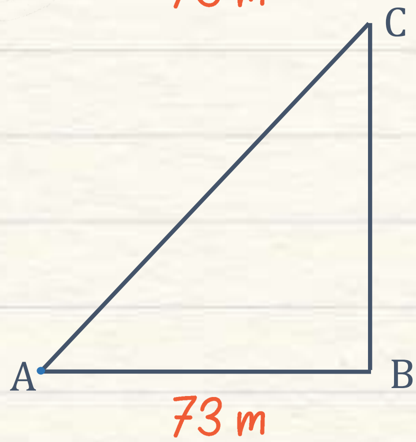
Step 1

Represent the 3D object by a vertical line.



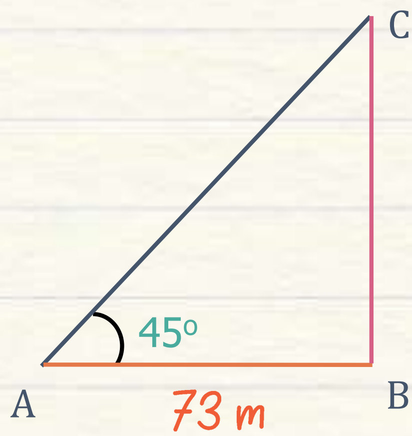
Step 2

Represent the observer as a point object.



Step 3

Label the angle, height, and distance.

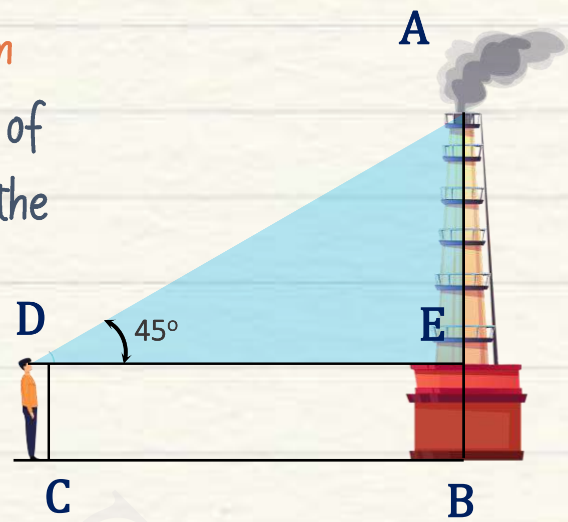


3. Trigonometric Ratios of Some Common Angles

Angles / Ratios	Logic	0°	30°	45°	60°	90°
sinθ	sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	Reverse sinθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	$\frac{\sin\theta}{\cos\theta}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecθ	$\frac{1}{\sin\theta}$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secθ	$\frac{1}{\cos\theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotθ	$\frac{1}{\tan\theta}$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

4. Method of Solving Questions

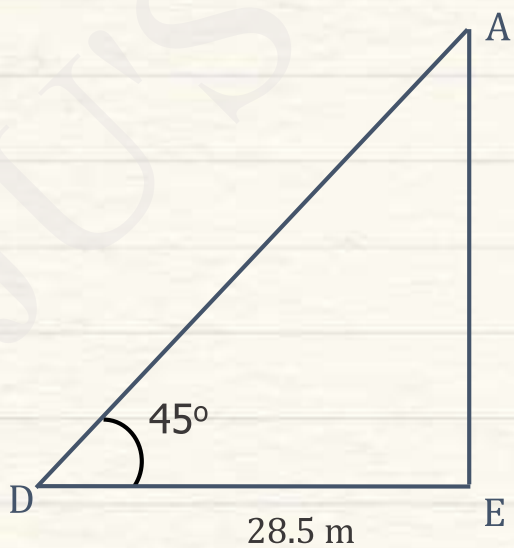
An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?



Steps to Draw the figure:

Step 1

Draw the figure correctly.



Step 2

Identify the unknown length.

AB = ?

Step 3

Use the relevant trigonometric ratios to find these lengths.

$$\tan 45^\circ = \frac{AE}{DE}$$

$$1 = \frac{AE}{28.5}$$

$$AE = 28.5 \text{ m}$$

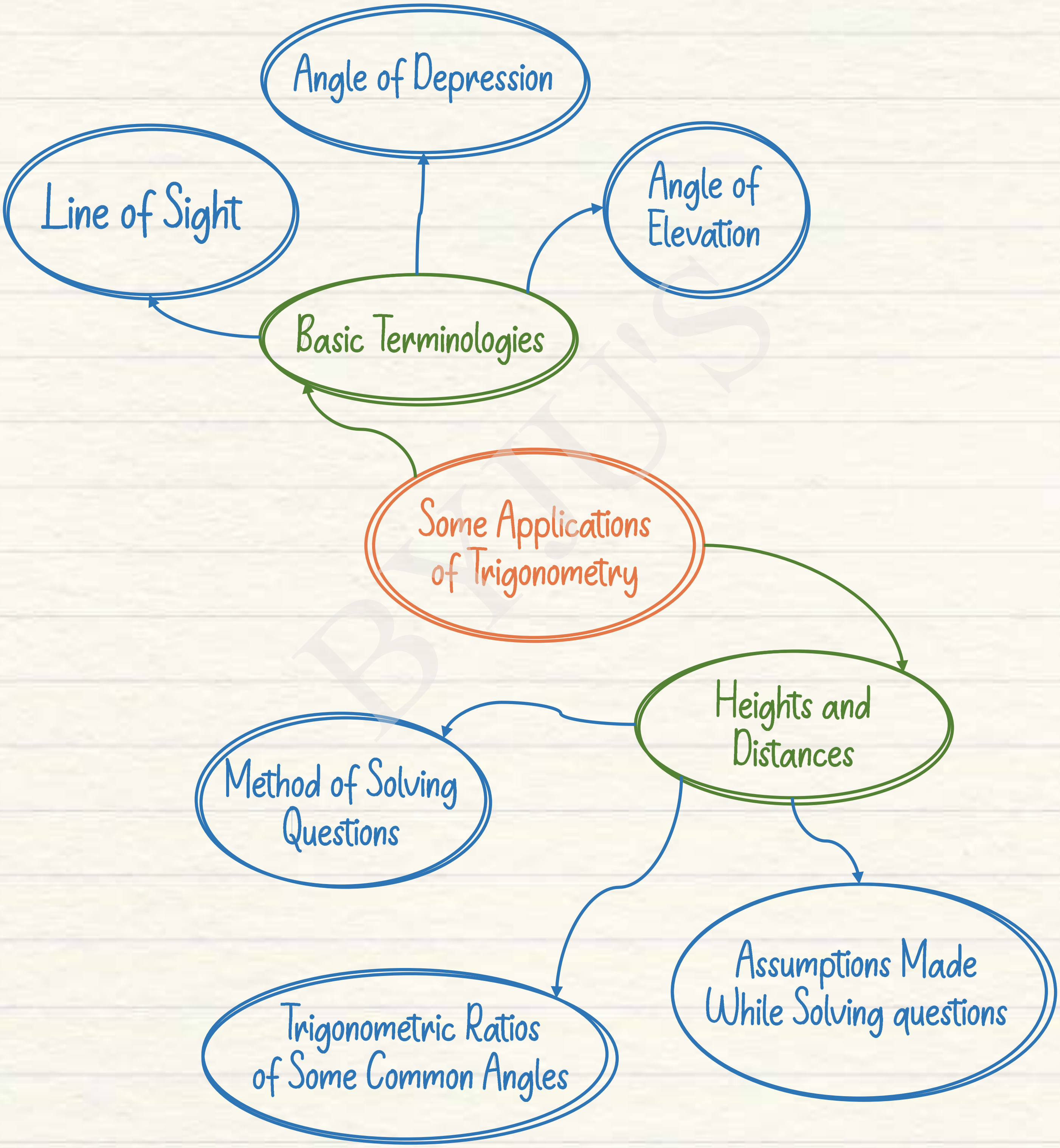
Step 4

Solve to find the unknown length

So, the height of the chimney

$$AB = (28.5 + 1.5) \text{ m} = 30 \text{ m.}$$

Mind Map



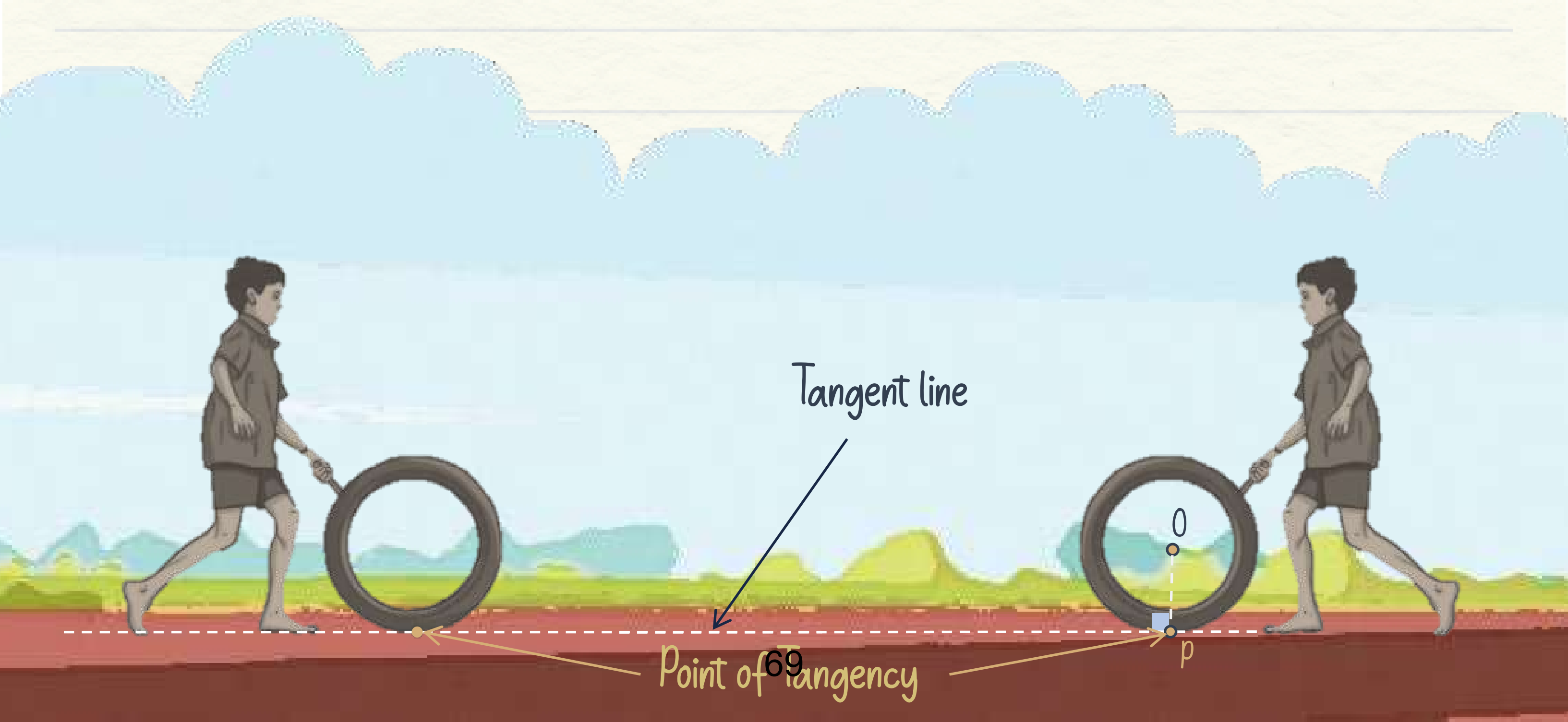
Angle of Depression
Line of Sight
Angle of Elevation
Basic Terminologies
Some Applications of Trigonometry
Heights and Distances
Method of Solving Questions
Assumptions Made While Solving questions
Trigonometric Ratios of Some Common Angles

BYJU'S CHAPTER NOTES

Circles

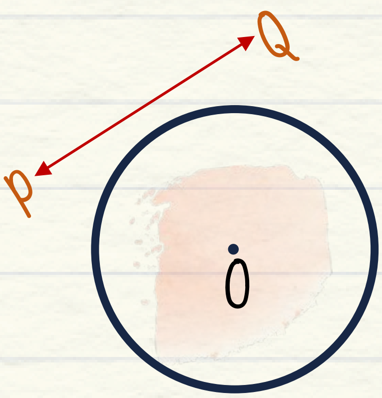
Topics

1. Lines related to a Circle
2. Tangents and Secants
3. Number of Tangents
4. Theorems related to a Tangent
5. Important Corollaries

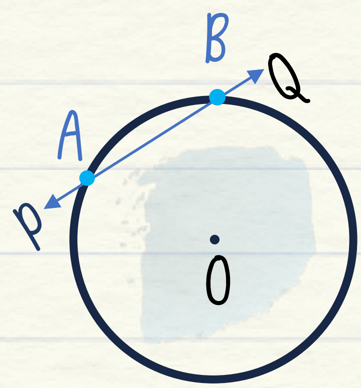


Lines related to Circle

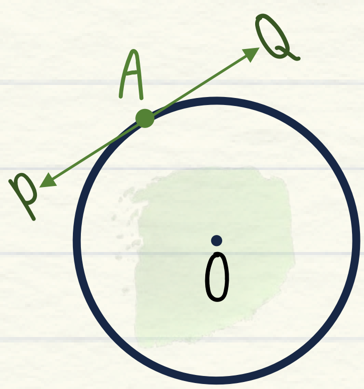
Tangents and Secants



* Does not touch the circle
* No point of intersection

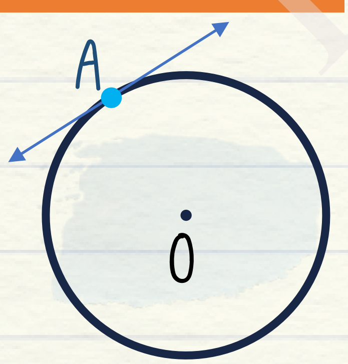


* 2 points of intersection
* PQ is the secant

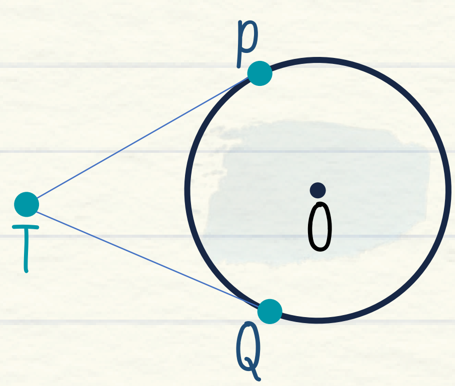


* Touches only at 1 point
* PQ is called tangent

No. of Tangents



For any point on the circumference of a circle,
No. of tangents = 1



No. of tangents from an external point to circle = 2

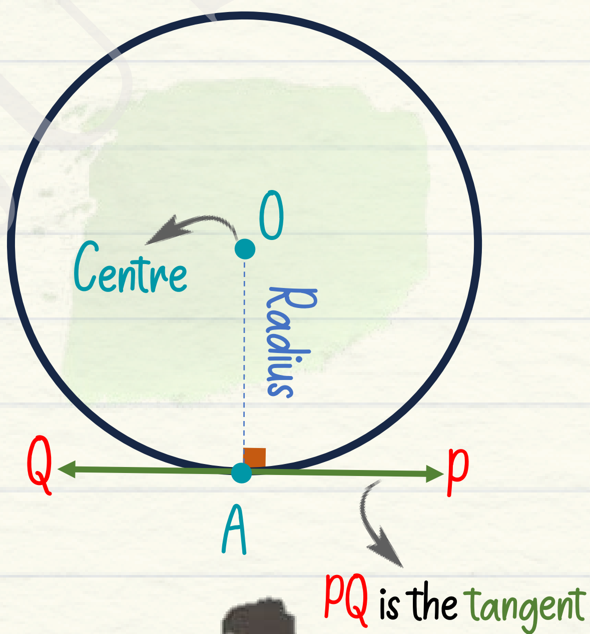
Theorems related to Tangent

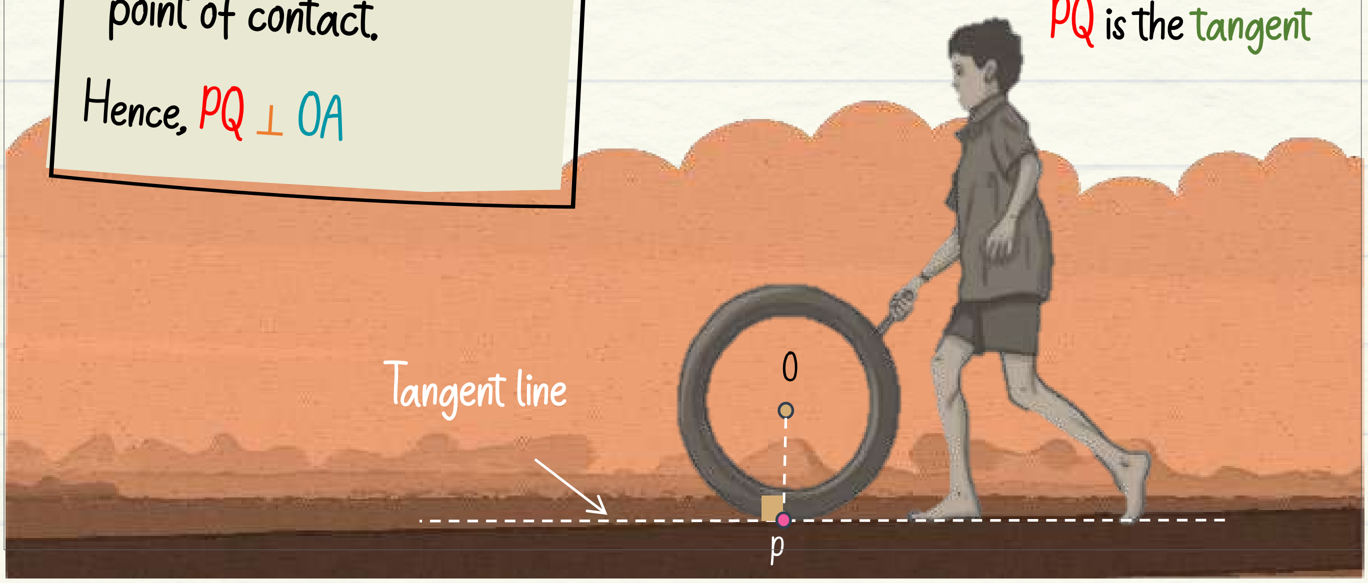
- **Theorem 1** — Tangents and Radius
- **Theorem 2** — Tangents from external point

1: Tangents and Radius

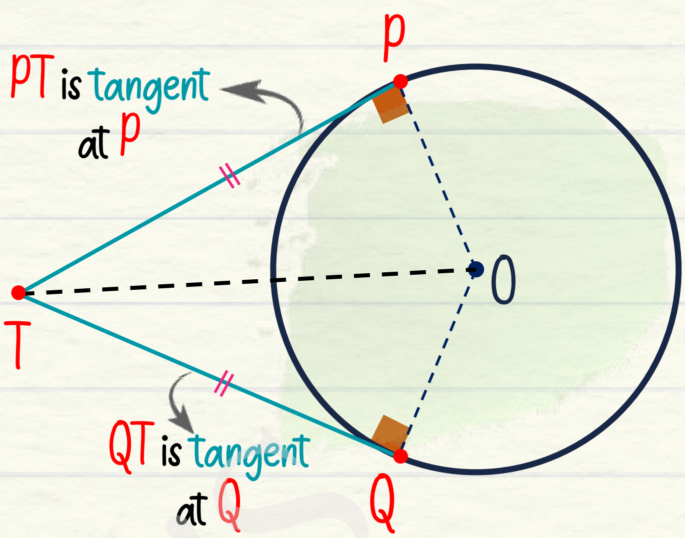
Theorem :– The tangent at any point of the circle is perpendicular to the radius through the point of contact.

Hence, $PQ \perp OA$





2: Tangents from external point

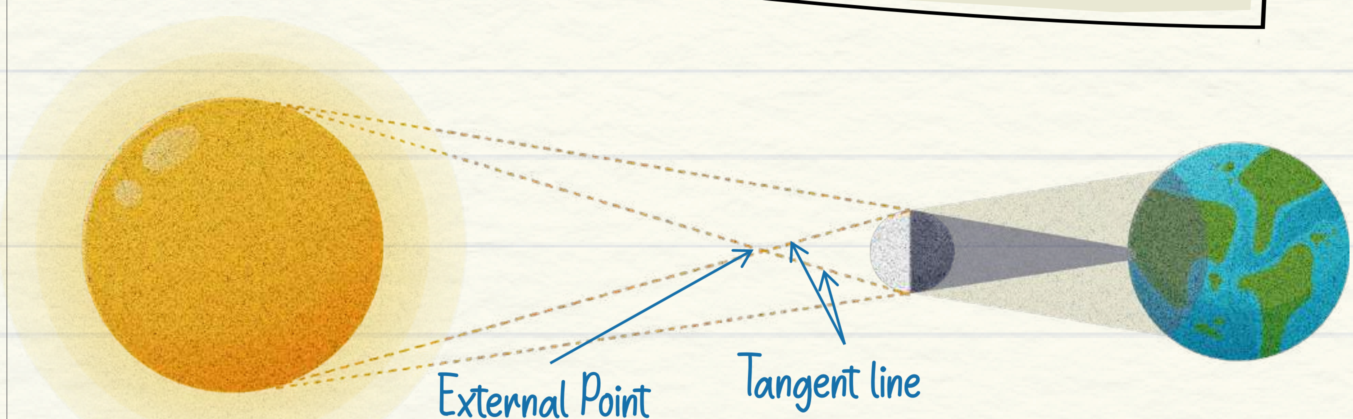


Theorem :- The lengths of tangents drawn from an external point to a circle are equal.

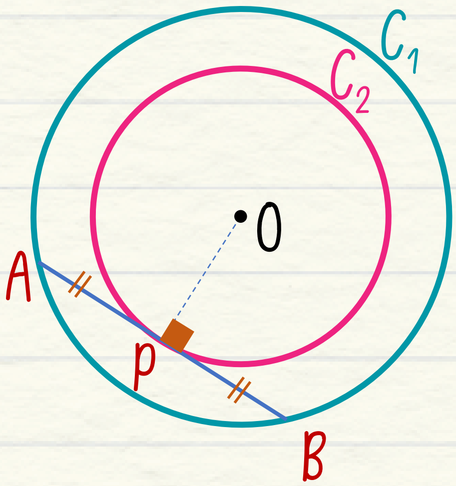
Hence, PT = QT

Can be proved in two ways :-

- Congruence of △TOP & △TOQ
- Pythagoras" theorem

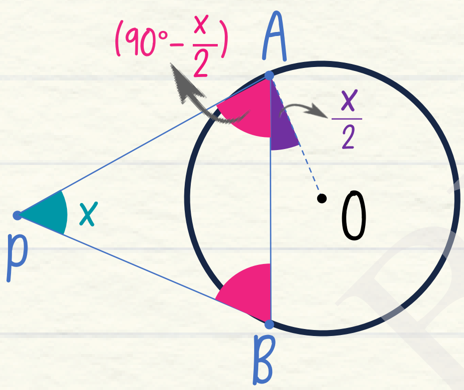


Important Corollaries



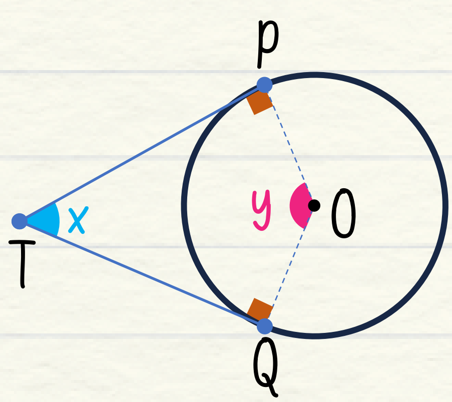
For C_1 and C_2 being concentric circles,

- OP is perpendicular bisector of AB
- AP = PB



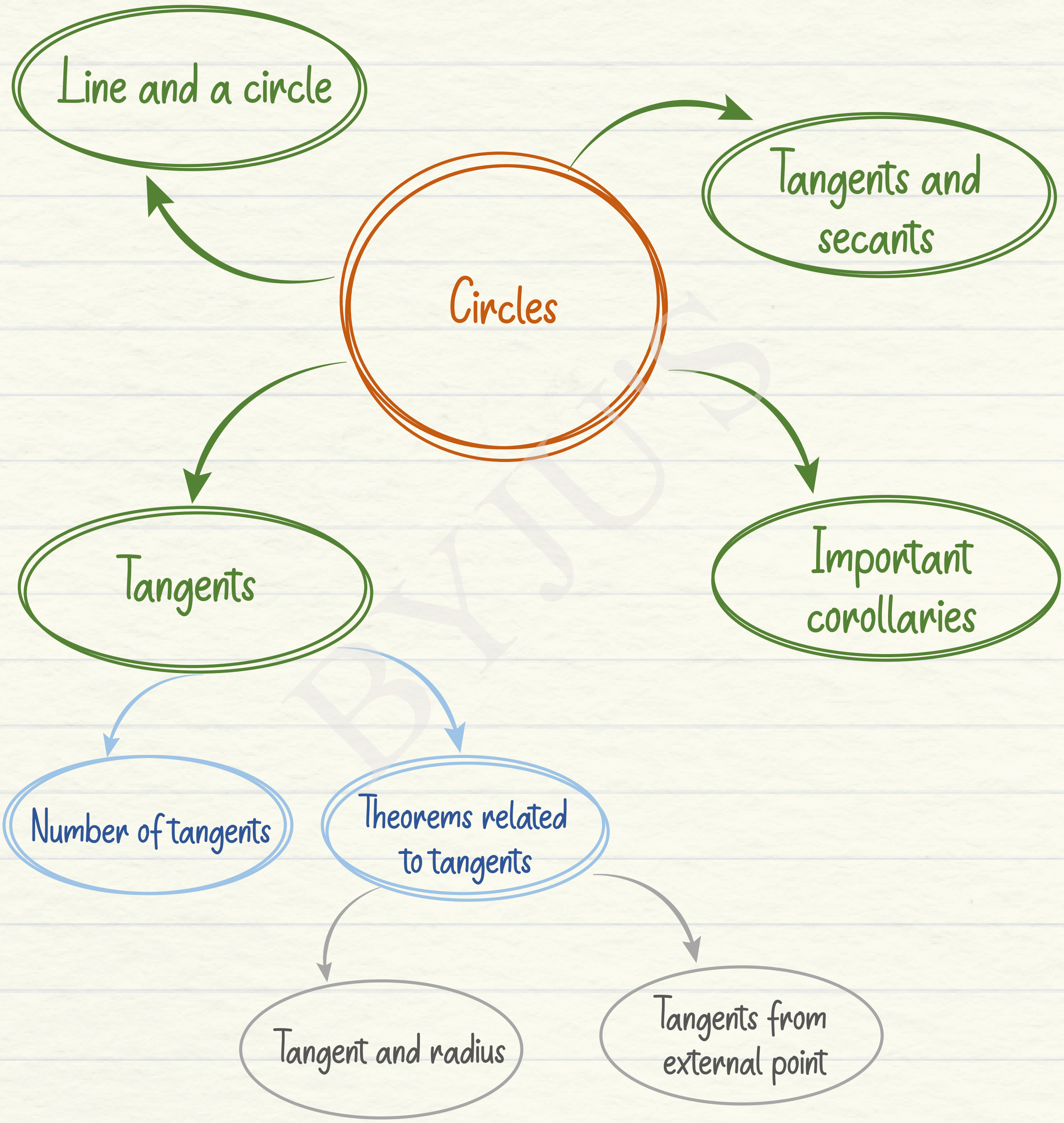
PA and PB are 2 tangents drawn from an external point P to a circle with centre at O,

- $\angle APB = 2\angle BAO$
- $\angle PAB = \angle PBA = (90° - \frac{x}{2})$



- x and y are supplementary
  i.e. $x + y = 180°$

Mind Map


Circles
Line and a circle
Tangents and secants
Tangents
Important corollaries
Number of tangents
Theorems related to tangents
Tangent and radius
Tangents from external point

BYJU'S
CHAPTER NOTES

Areas Related to Circle

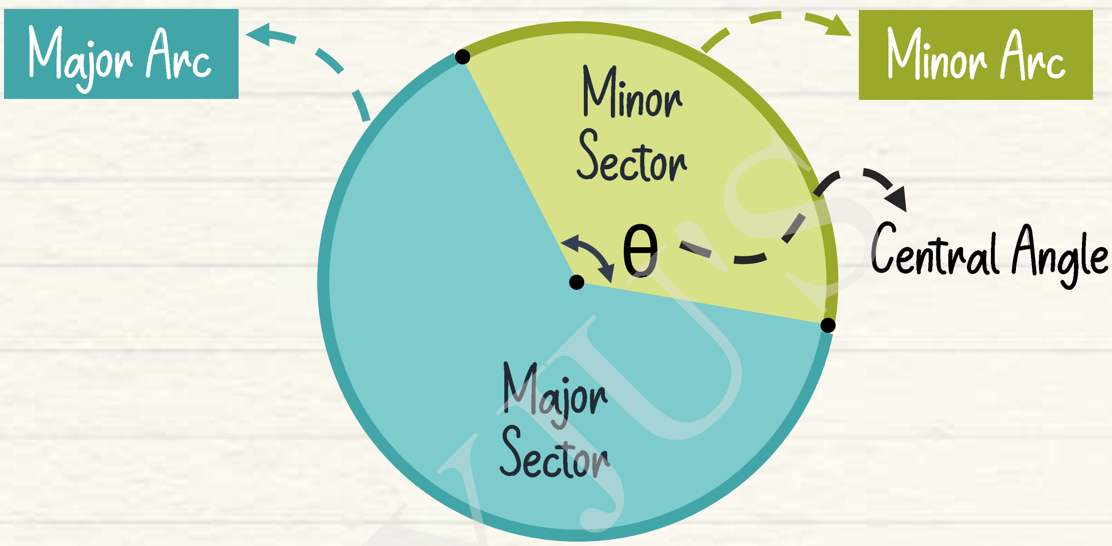
Topics

1. Area of sector
2. Area of segment
3. Area of combined plane figures

1. Area of Sector

Sector

A sector of a circle is the portion of an area enclosed by two radii and an arc.



Area of minor sector

$$\frac{\theta}{360°} \times \pi r^2$$

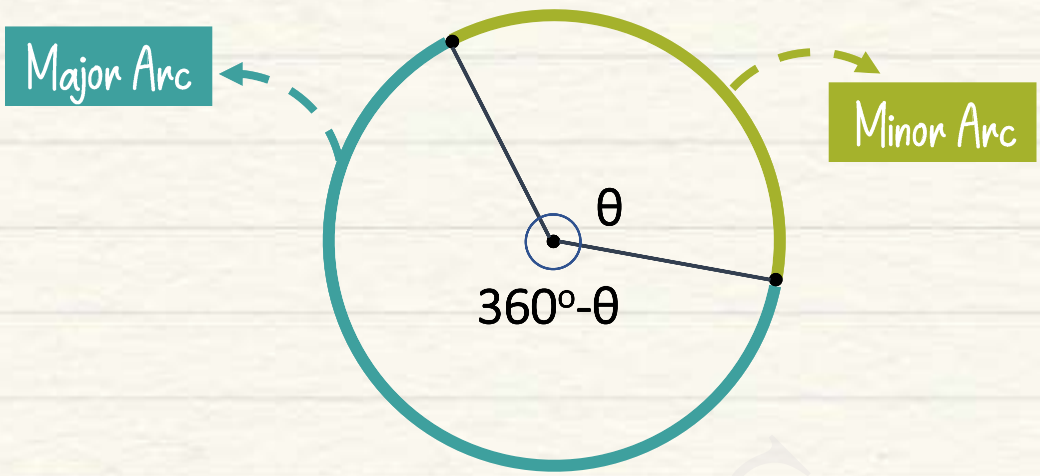
Area of major sector

$$\frac{360° - \theta}{360°} \times \pi r^2$$

Central angle θ must be in degrees.

If θ is given in radians, multiply it with $\frac{180^0}{\pi}$ to convert in degrees.

Length of Arc



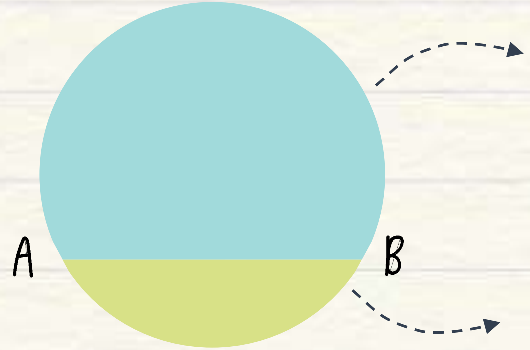
Length of minor arc = $\frac{\theta}{360°} \times 2\pi r$

Length of major arc = $\frac{360° - \theta}{360°} \times 2\pi r$

2. Area of Segment

Segment

A segment of a circle can be defined as a region bounded by a chord and a corresponding arc lying between the chord's endpoints.



Segment corresponding to major arc called **major segment.**

Segment corresponding to minor arc called **minor segment.**

Area of Segment

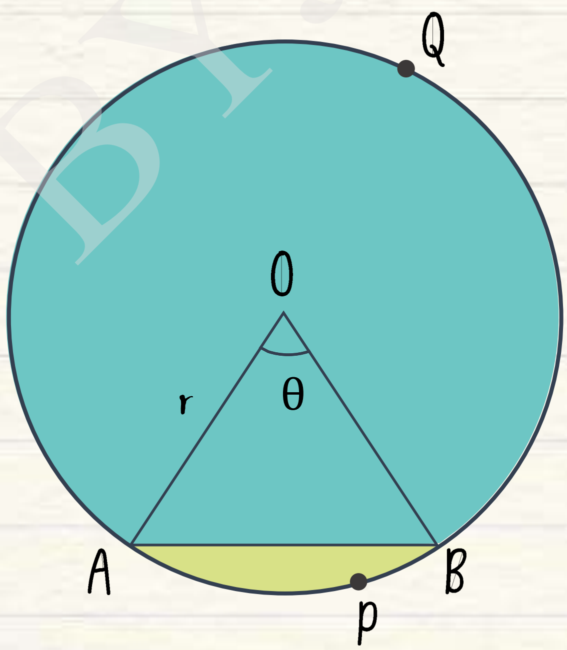
When θ is given in degrees,

Area of a segment = $\left(\frac{1}{2}\right) \times r^2 \times [\left(\frac{\pi}{180^0}\right)\theta - sin\theta]$

When θ is given in radians,

Area of a segment = $\left(\frac{1}{2}\right) \times r^2[\ \theta\ - sin\theta]$

Area of major segment = Area of sector OAQB + Area of ΔOAB



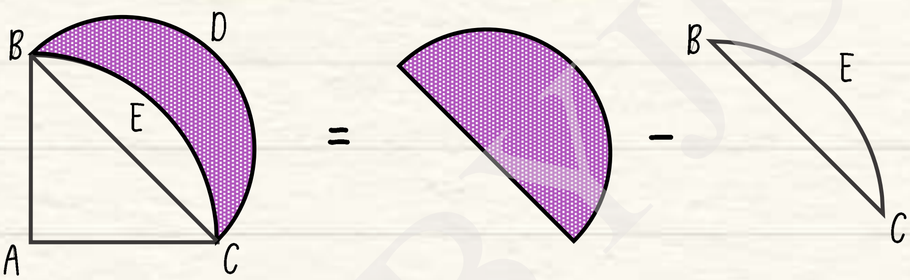
Area of minor segment= Area of the sector OAPB – Area of ΔOAB

3. Area of Combined Plane Figures

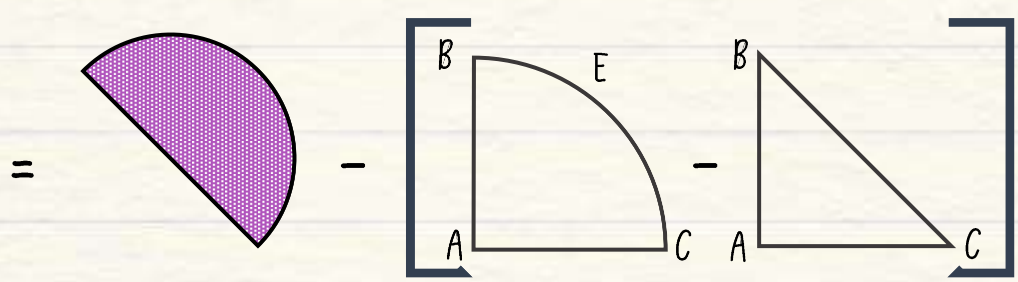
General Formula

Areas of shaded region = Area of entire figure – Area of non shaded region

Example

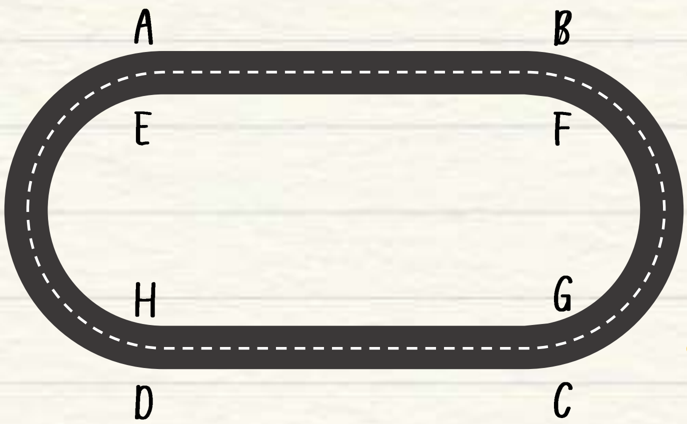


Area of shaded figure = Area of semicircle – Area of minor segment



= Area of semicircle – (Area of sector ABEC – Area of ΔABC)

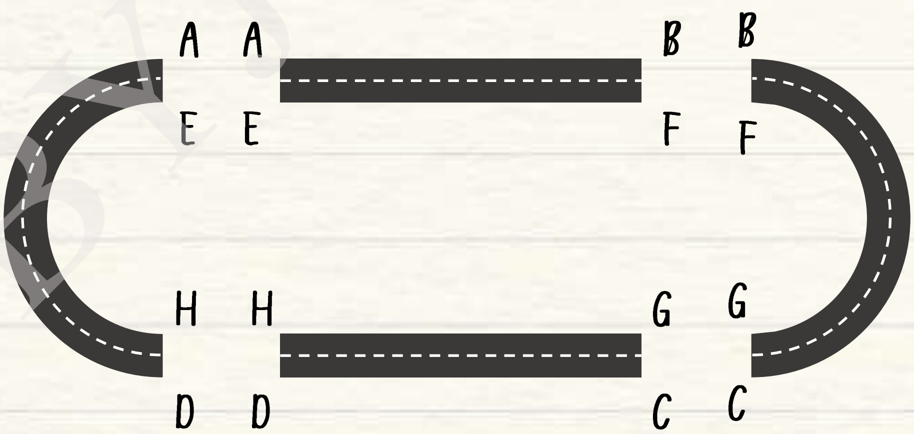
Find the area of the track.



Methodology

Step 1

Simplify the given figure into known standard shapes.



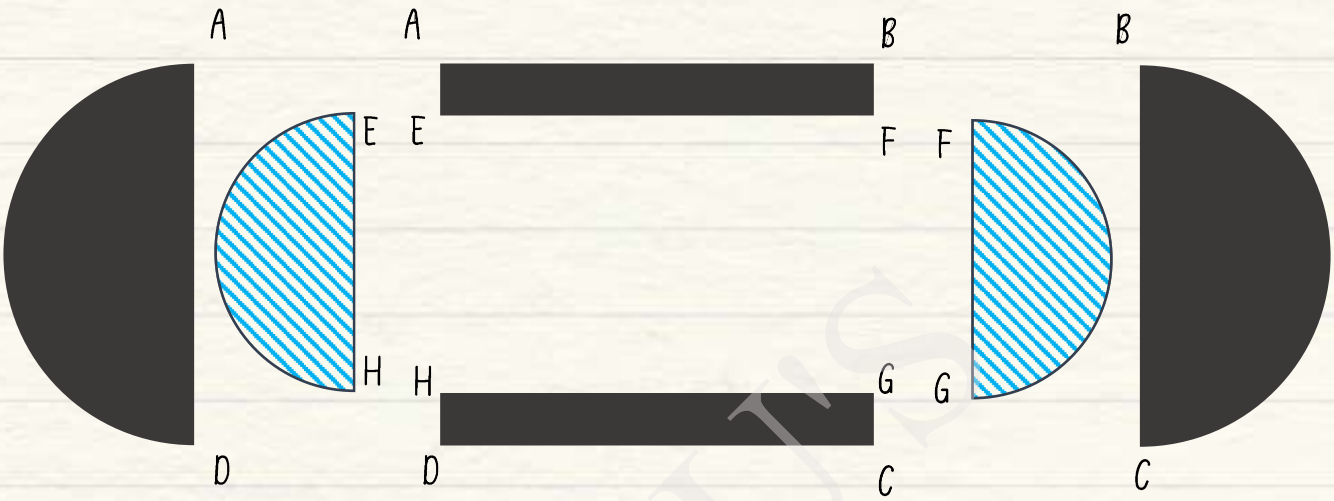
Step 2

Apply the formula of area on each shape.

= Area of rectangle ABFE + Area of rectangle HGCD
+ Area of the sidetracks

Step 3

To find the area of the required region, add or subtract the areas of the standard figures as per the requirement.



= Area of rectangle ABFE + Area of rectangle HGCD

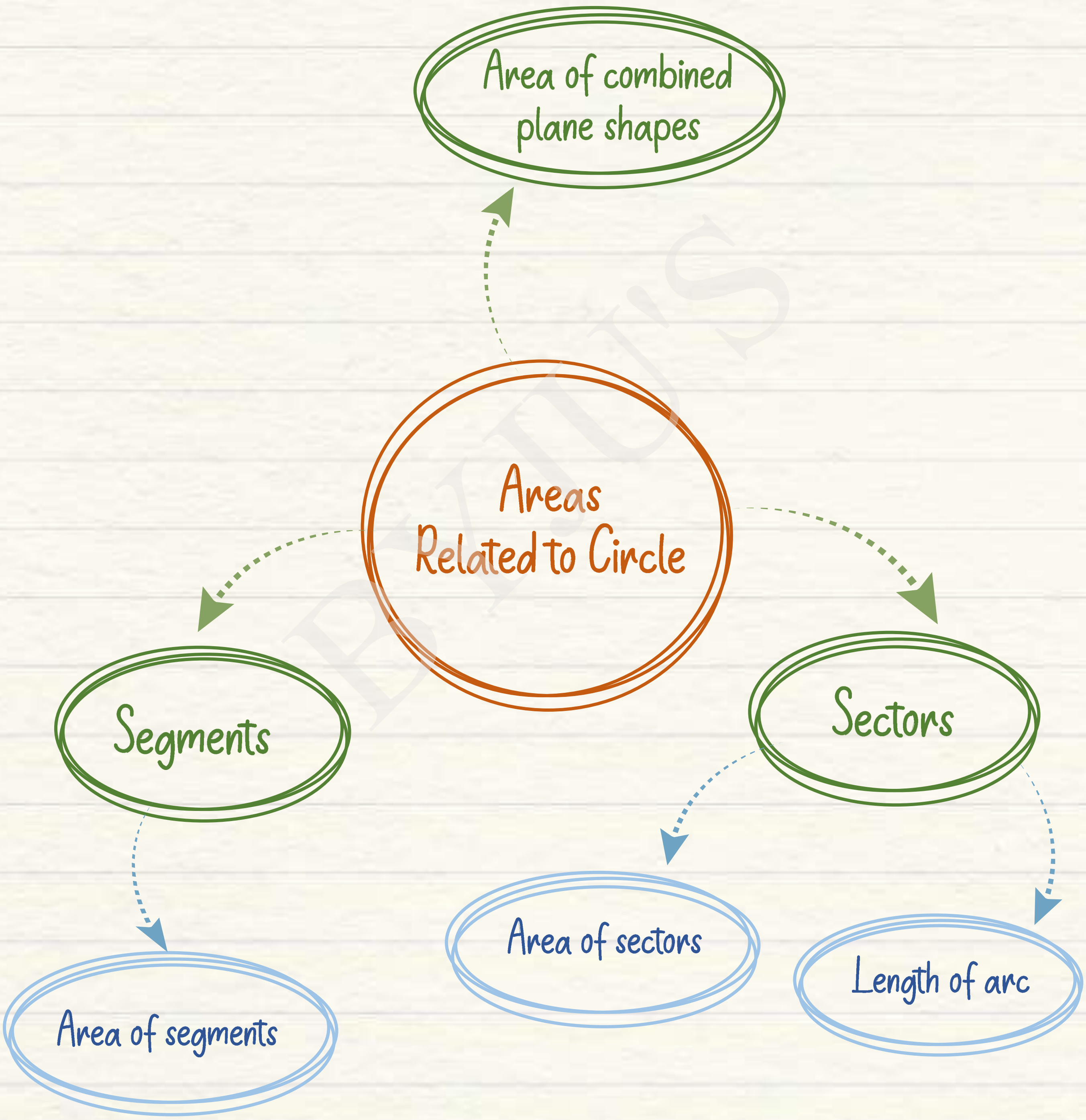
+

(Area of semicircle with diameter AD − Area of semicircle with diameter EH)

+

(Area of semicircle with diameter BC − Area of semicircle with diameter FG)

Mind Map


Area of combined plane shapes
Areas Related to Circle
Segments
Sectors
Area of segments
Area of sectors
Length of arc

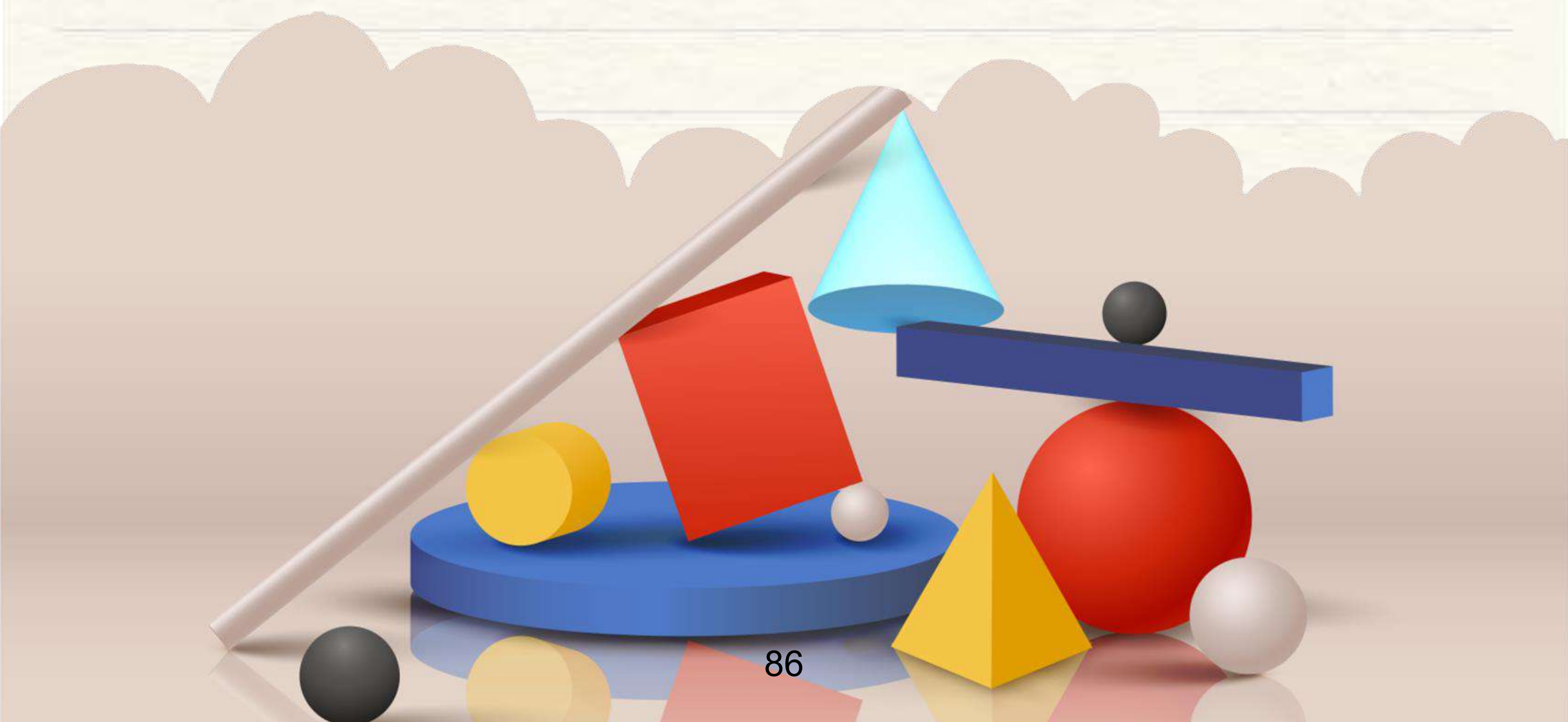
MATHEMATICS

BYJU'S CHAPTER NOTES

Surface Areas and Volumes

Topics to be Covered

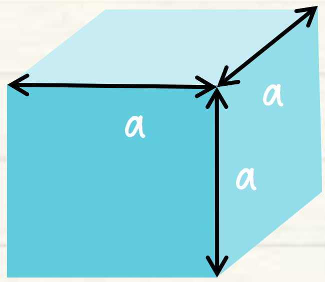
1. Formulae of Solids
2. Combination of Solids
3. Surface Area of Combined Solids
4. Volume of Combined Solids
5. Conversion of Solids



1. Formulae of Solids

Here are surface areas and volumes of few solids before we look at combined solids.

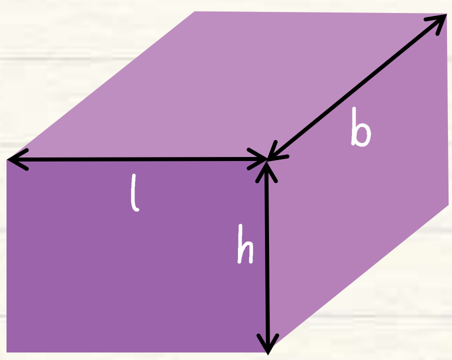
Cube



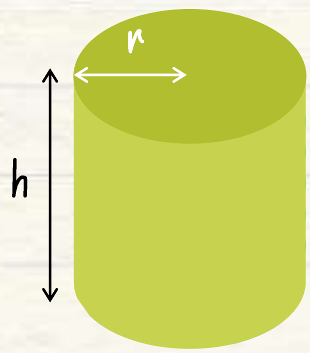
$4a^2$	: Lateral surface area
$6a^2$	: Total surface area
a^3	: Volume

Cuboid

Lateral surface area :	$2h(l + b)$
Total surface area :	$2(lb + bh + hl)$
Volume :	lbh



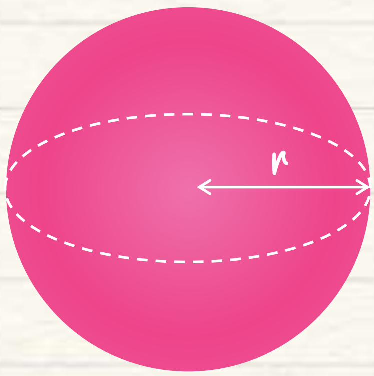
Cylinder



$2\pi rh$	: Curved surface area
$2\pi rh + 2\pi r^2$	: Total surface area
$\pi r^2 h$	: Volume

1. Formulae of Solids

Sphere



$4\pi r^2$: Curved surface area

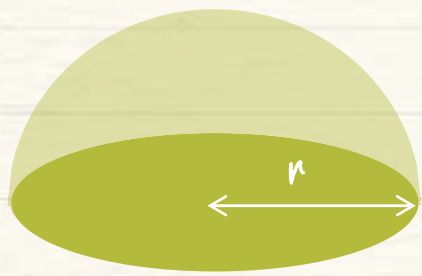
$\frac{4}{3}\pi r^3$: Volume

Hemisphere

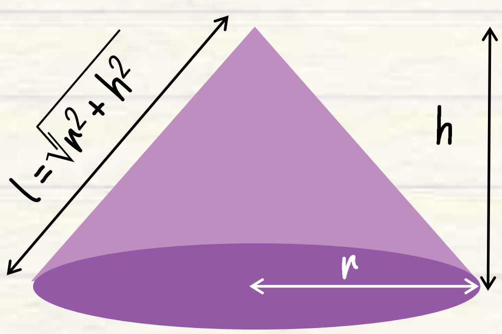
Curved surface area : $2\pi r^2$

Total surface area : $3\pi r^2$

Volume : $\frac{2}{3}\pi r^3$



Cone



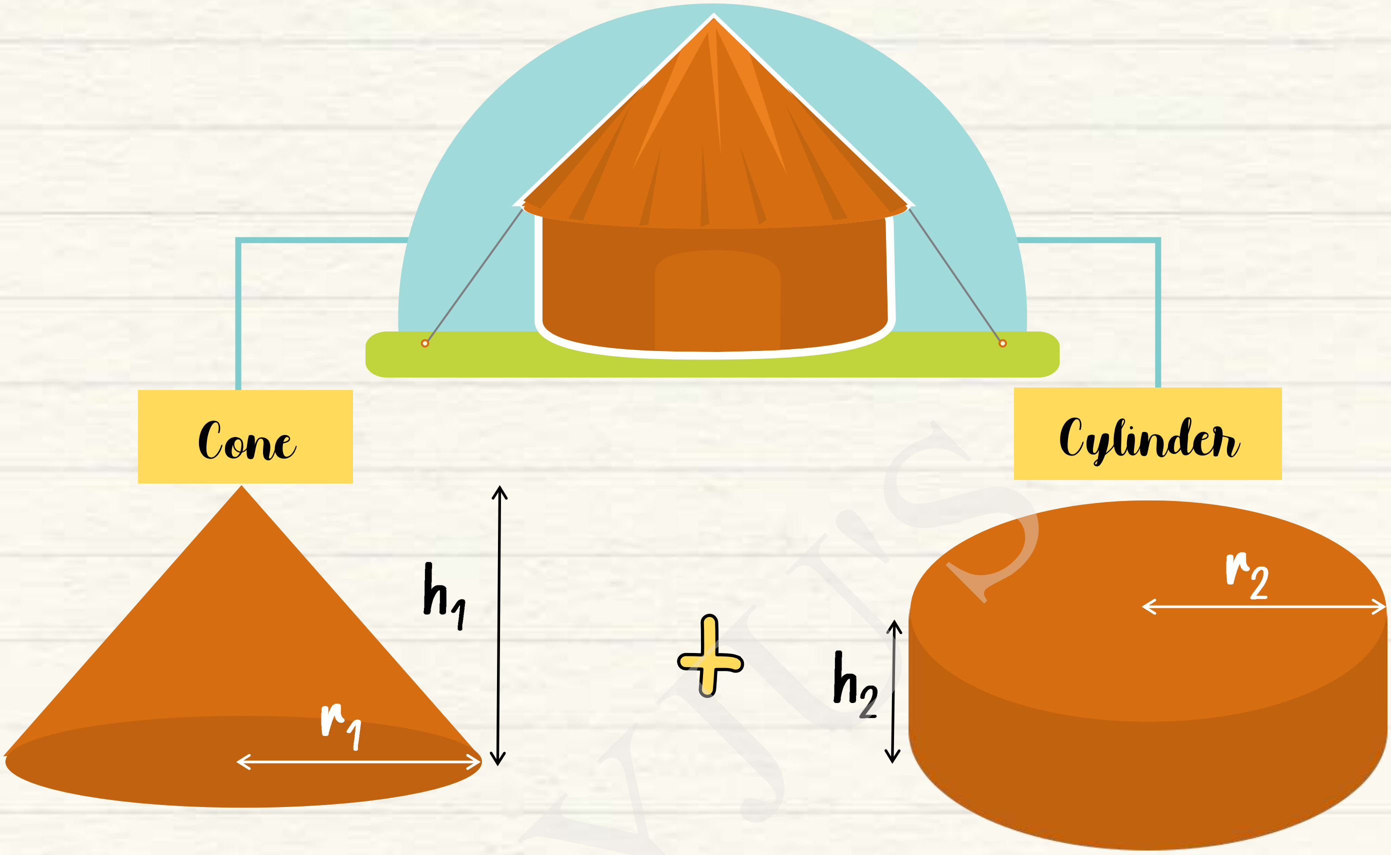
πrl : Curved surface area

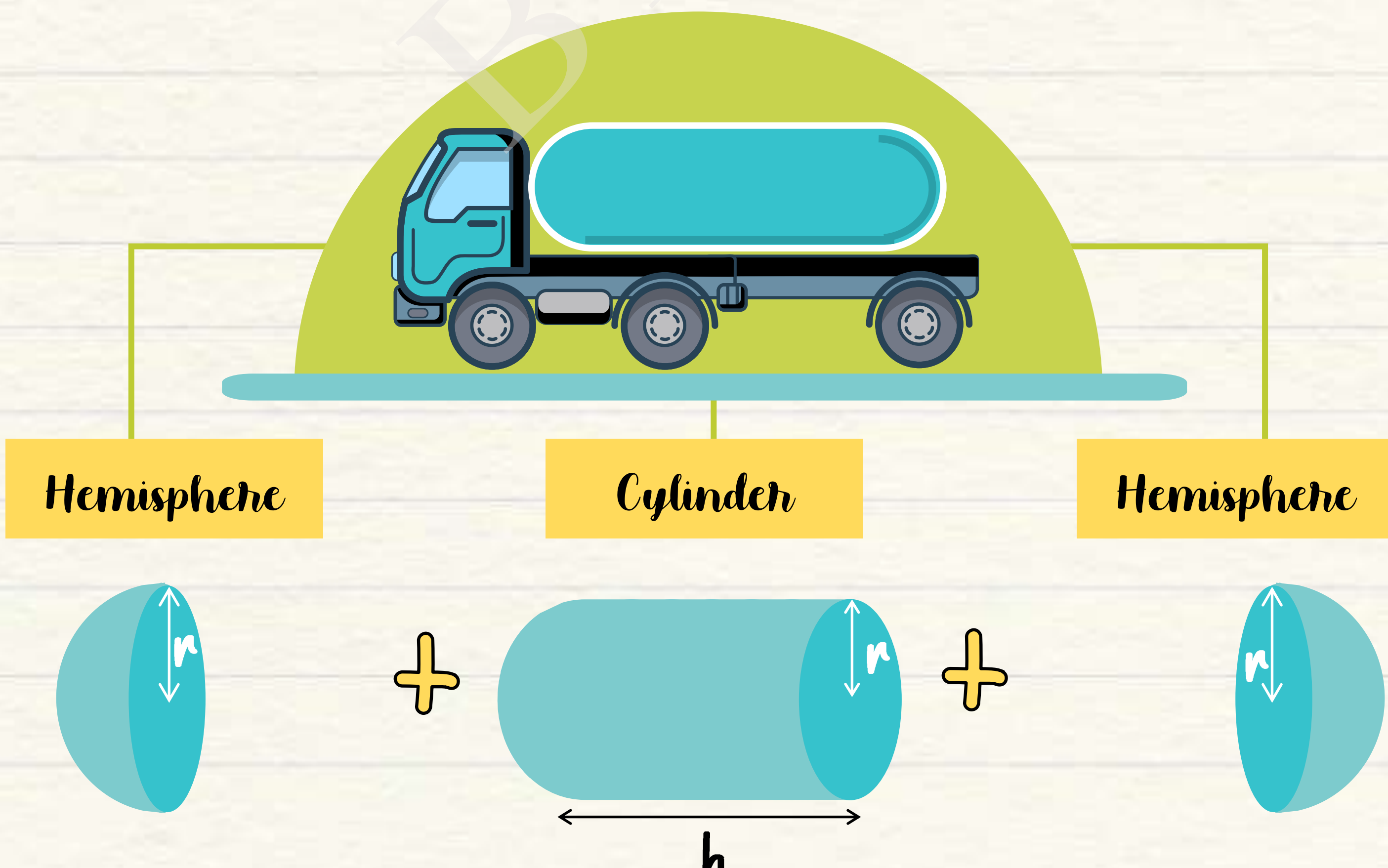
$\pi rl + \pi r^2$: Total surface area

$\frac{1}{3}\pi r^2 h$: Volume

2. Combination of Solids

Shapes that are formed by combining two or more solids.

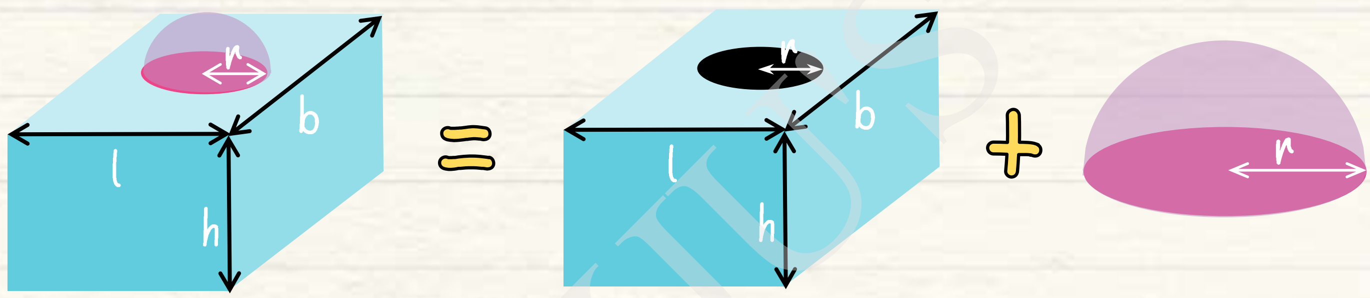




3. Surface Area of Combination of Solids

It is the sum of the surface areas of individual solid"s visible portion, in the given combined solid.

Total Surface Area



Total surface area of the shape

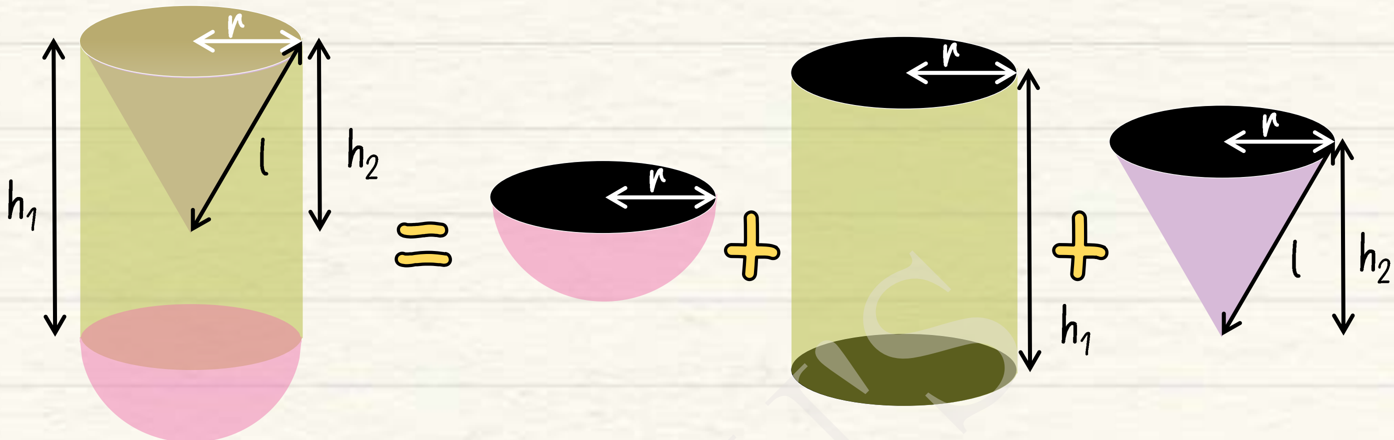
=

{ Total surface area of cuboid
+ Curved surface area of hemisphere
− Base area of hemisphere }

$$2(lb + bh + hl) + 2\pi r^2 - \pi r^2$$

3. Surface Area of Combination of Solids

Total Surface Area



Total surface area of the shape

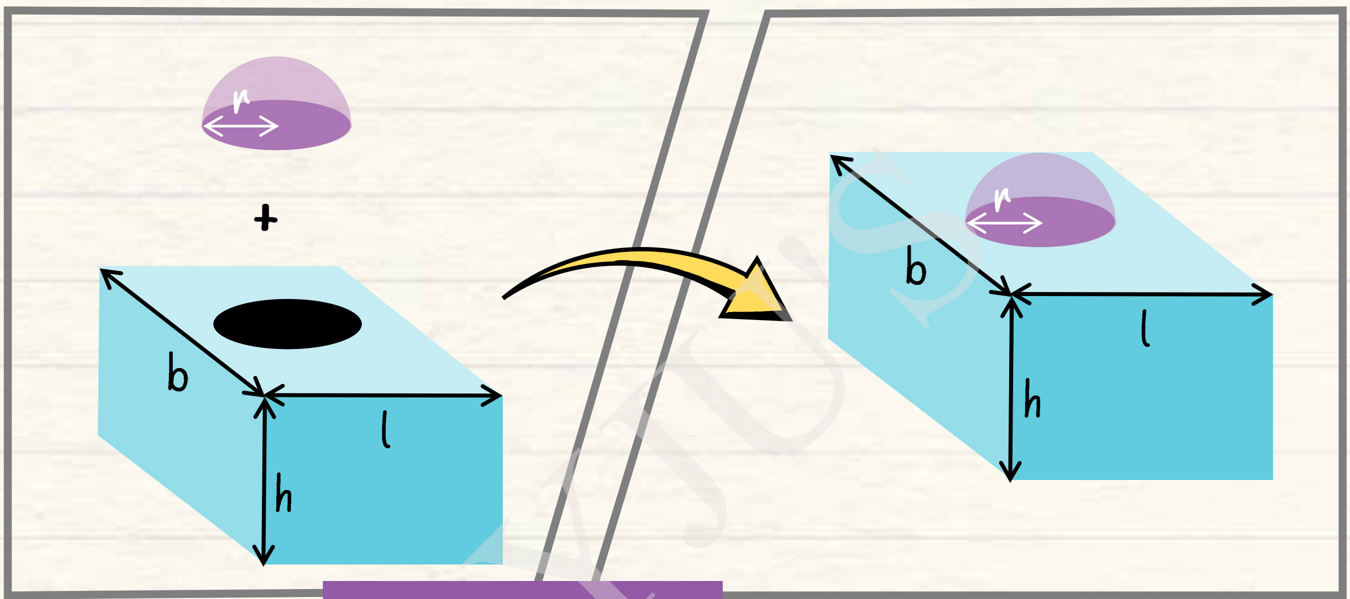
=

Curved surface area of hemisphere
+ Curved surface area of cylinder
+ Curved surface area of cone

$$2\pi r h_1 + \pi r l + 2\pi r^2$$

4. Volume of Combination of Solids

It is the sum of the volumes of solids that are being combined, and subtraction of the volumes of the solids that are being removed.



Volume

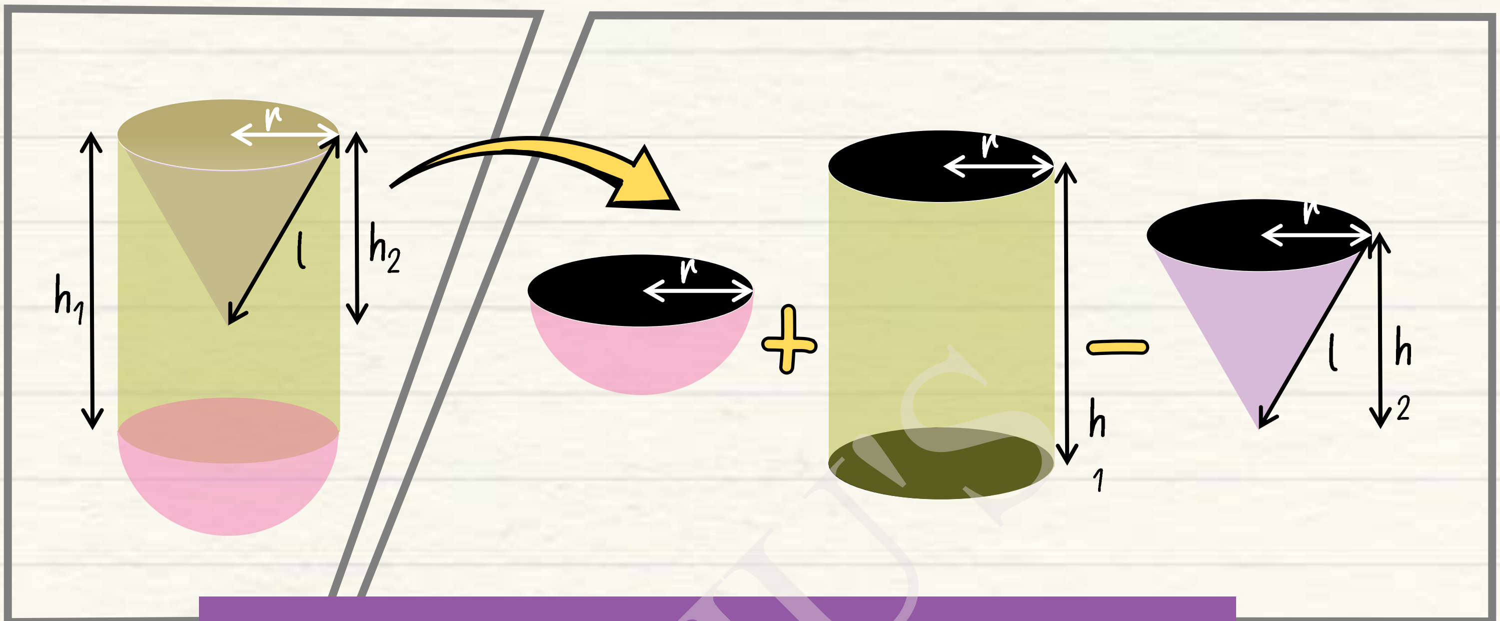
Volume of cuboid

+

Volume of hemisphere

$$lbh + \frac{2}{3}\pi r^3$$

4. Volume of Combination of Solids



Volume

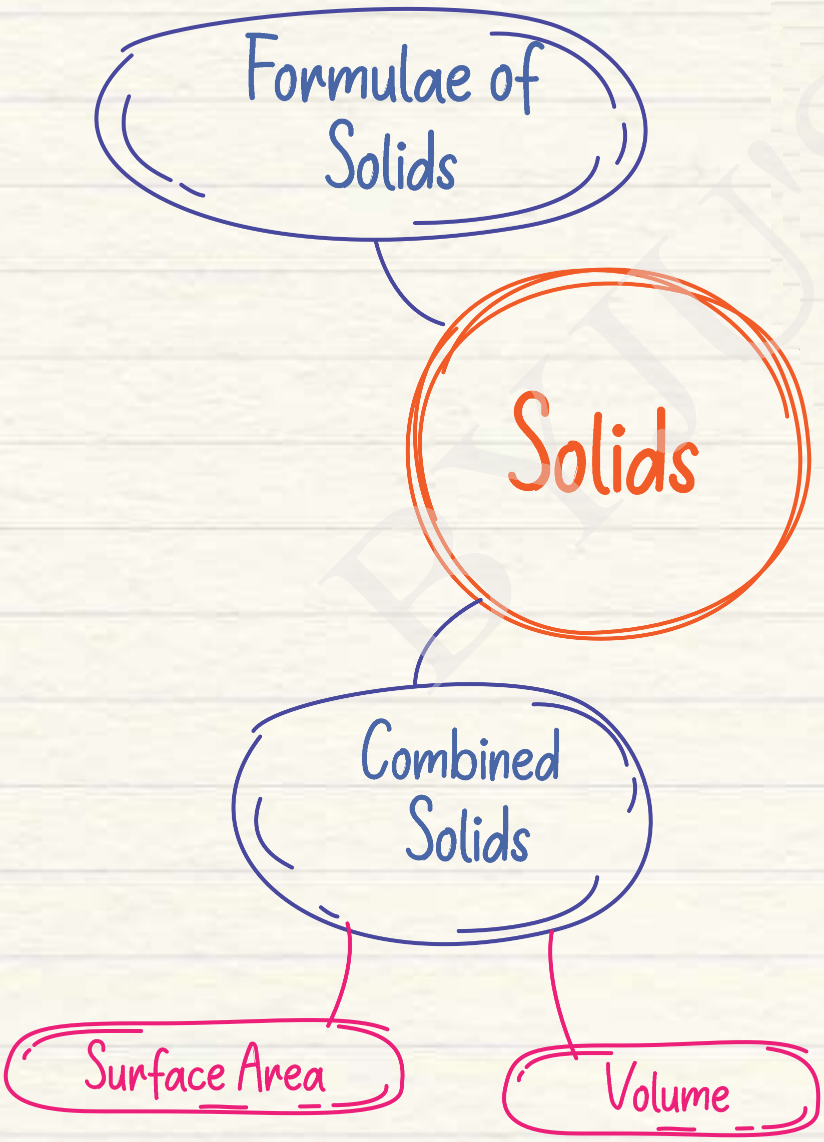
Volume of the shape

=

{ Volume of Cylinder
+ Volume of hemisphere
− Volume of cone }

$$\pi r^2 h_1 - \frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3$$

Mind Map


Formulae of Solids
Solids
Combined Solids
Surface Area
Volume

BYJU'S CHAPTER NOTES

Statistics

Topics

1. Mean
2. Cumulative Frequency
3. Median
4. Mode

Mean of Grouped Data

Mean

Mean is a measure of central tendency which gives the average of a data.

Direct Method

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

Class mark (x_i) = $\frac{\text{Upper Class Limit + Lower Class Limit}}{2}$

Assumed Mean Method

An arbitrary mean 'a' is chosen which is called 'assumed mean', somewhere in the middle of all the values of x.

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

Where $d_i = (x_i - a)$

Step Deviation Method

$$\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$$

Where $u_i = \frac{d_i}{h}$ and h is class size of class interval

Cumulative Frequency

Cumulative frequency is the sum of all the frequencies up to the current point.

Less-than type cumulative frequency table

Marks	Number of students
0–10	5
10–20	3
20–30	4
30–40	3

Marks	Cumulative frequency
Less than 10	5
Less than 20	5 + 3 = 8
Less than 30	8 + 4 = 12
Less than 40	12 + 3 = 15

More-than type cumulative frequency table

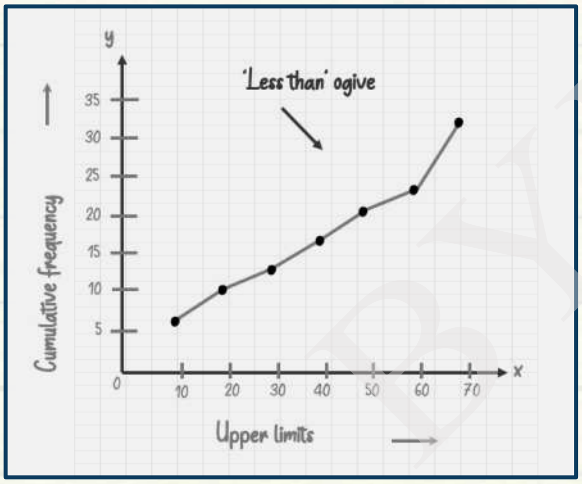
Marks	Number of students
0–10	5
10–20	3
20–30	4
30–40	3

Marks	Cumulative frequency
More than or equal to 0	5
More than or equal to 10	15 – 5 = 10
More than or equal to 20	10 – 3 = 7
More than or equal to 30	7 – 4 = 3

Graphical Representation of Cumulative Frequency Distribution

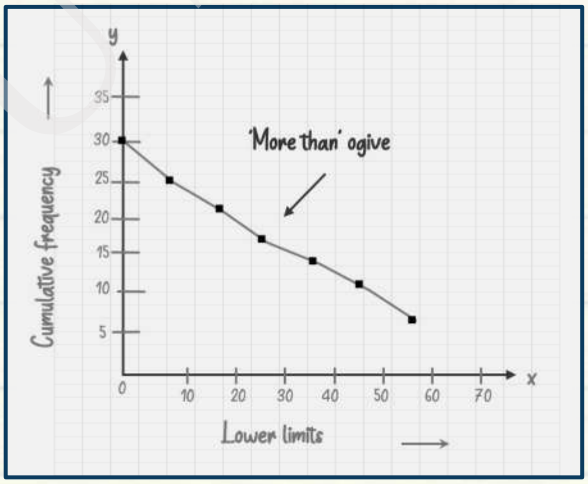
Less than Ogive

To draw the graph of less than ogive, take the upper limits of the class interval and mark the respective less than frequency. Then, join the dots by a smooth curve.



More than Ogive

To draw the graph of more than ogive, take the lower limits of the class interval on the x-axis and mark the respective more than frequency. Then, join the dots by a smooth curve.



Let's say class interval 70 – 80, the frequencies included in this interval are from $70 \leq f < 80$,
which means the frequencies corresponding to 80 do not belong to this class interval.

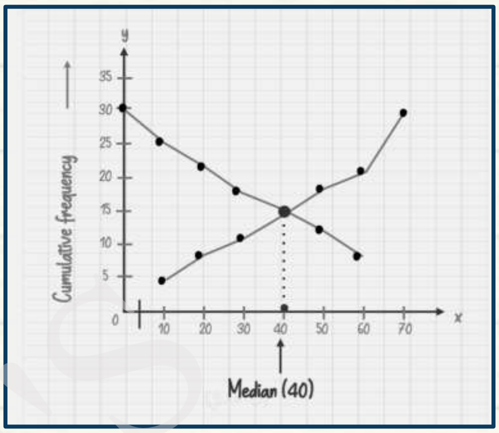
Median of Grouped Data

Algebraic Method

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

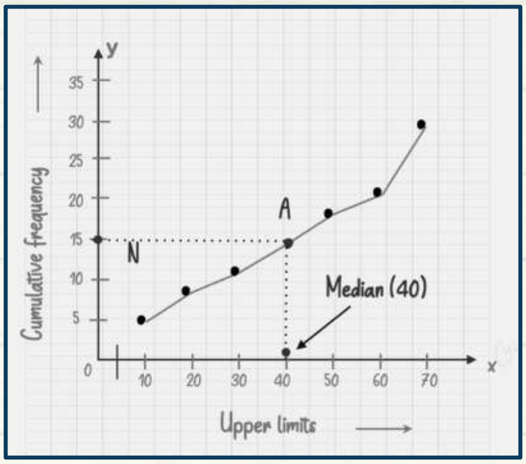
l = Lower limit of median class
n = Number of observations
f = Frequency of median class
cf = Cumulative frequency of preceding class
h = Class size

Graphical Method



Median can be obtained by either the less than type or more than type ogive. The given methodology is applicable for both, i.e., less than or more than ogive.

1. Find the middle point of total number of cumulative frequency of the given dataset and mark it as N on the y-axis.
2. From N, draw a line parallel to X axis to intersect the ogive at point A.
3. Drop a perpendicular from A on X axis. This value will represent the median.



Mode of Grouped Data

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

l = lower class limit of the modal class

h = class interval size

f_1 = frequency of the modal class

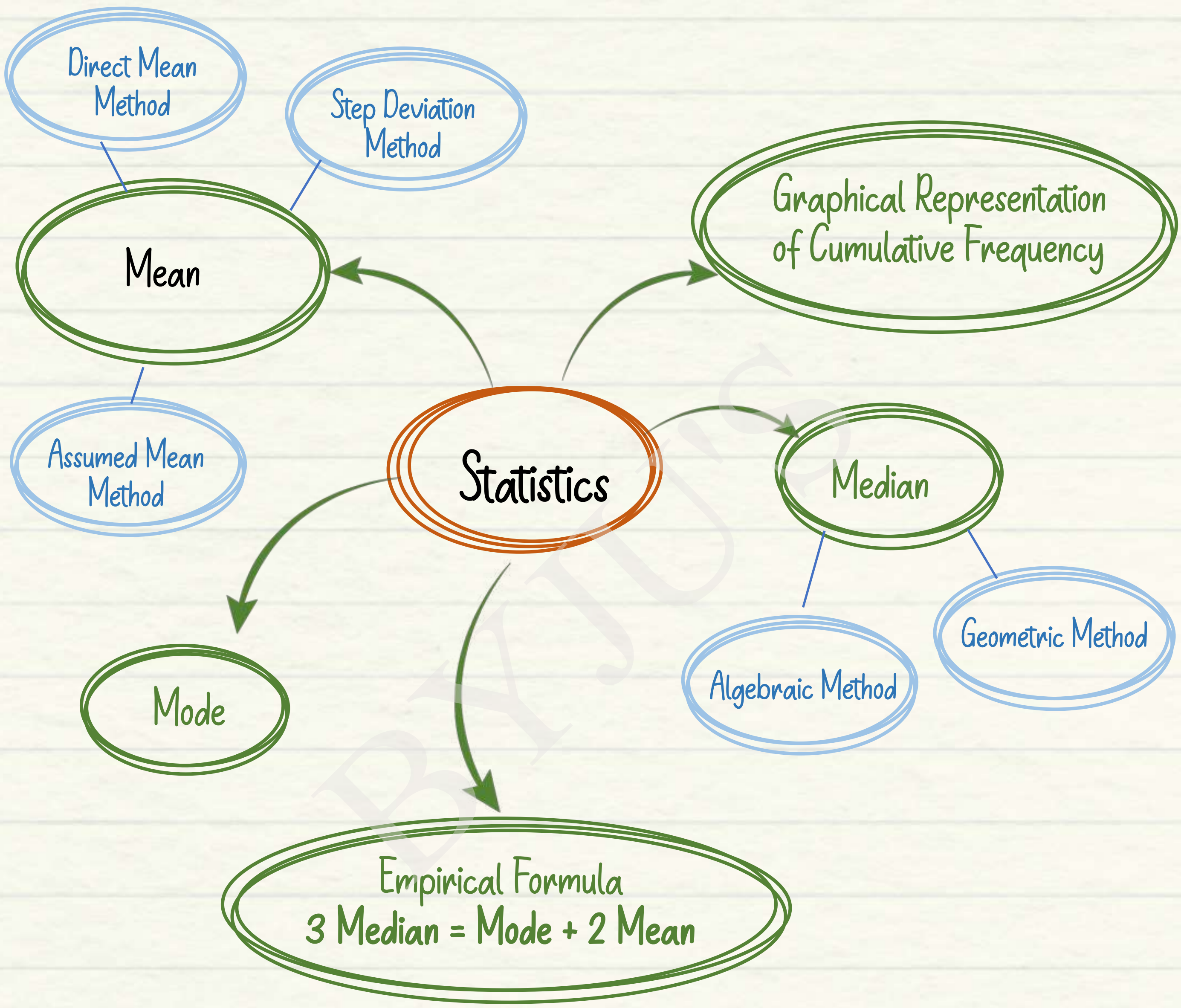
f_0 = frequency of the preceding class

f_2 = frequency of the succeeding class

Empirical Formula

$$3\ \text{Median} = \text{Mode} + 2\ \text{Mean}$$

Mind Map



Direct Mean Method
Step Deviation Method
Mean
Graphical Representation of Cumulative Frequency
Assumed Mean Method
Statistics
Median
Geometric Method
Algebraic Method
Mode
Empirical Formula
3 Median = Mode + 2 Mean

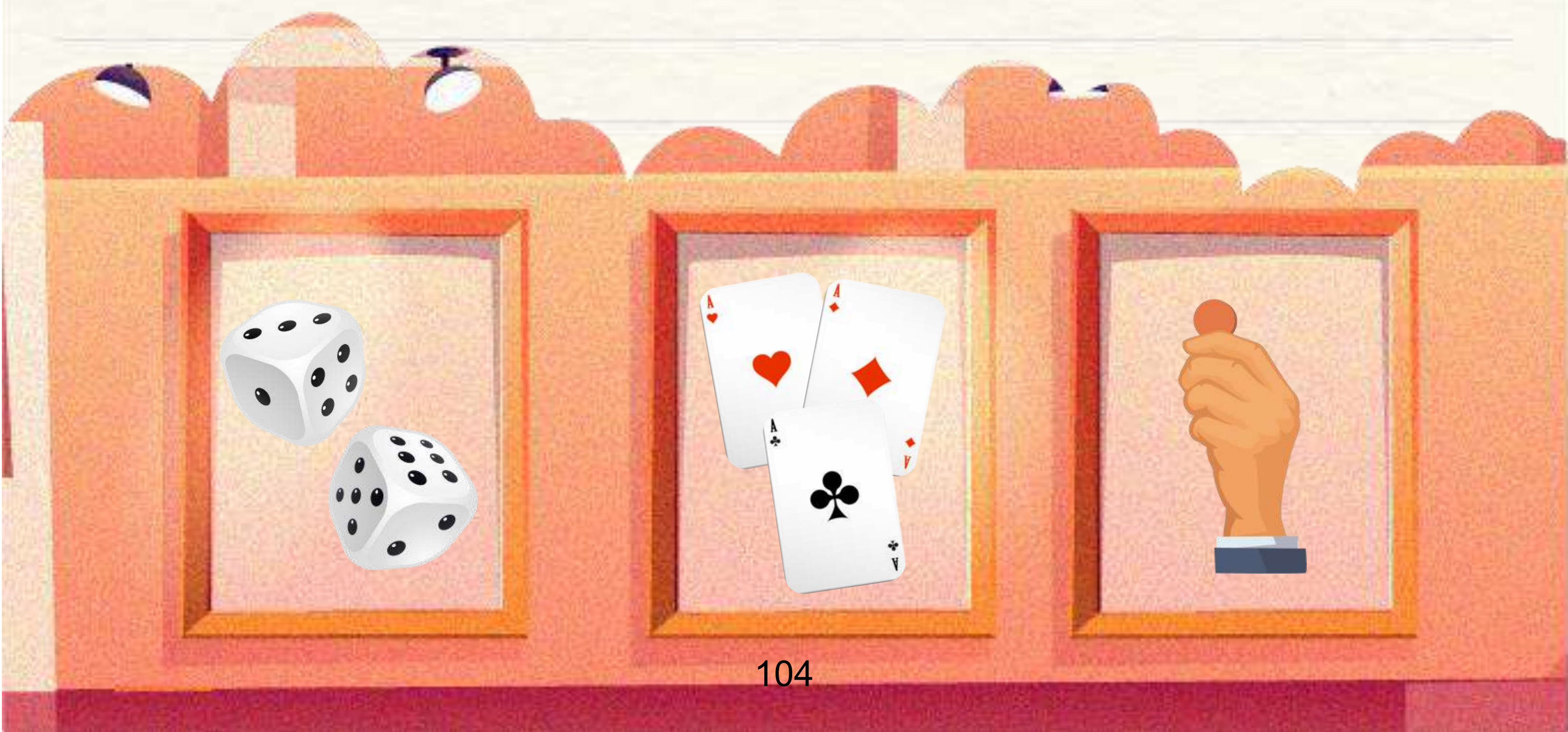
MATHEMATICS

BYJU'S CHAPTER NOTES

PROBABILTY

Topics

1. Basic Terminology
2. Types of Probability
   - 2.1 Theoretical Probability
3. Types of Events
4. Important Formulae



1. Basic Terminology

Random Experiment

- Has more than one possible outcomes.
- It is impossible to predict any outcome in advance.
- Examples:

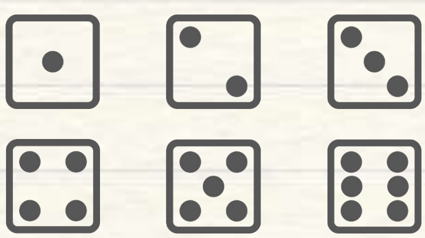
Tossing a coin

Rolling a dice

Drawing a card from a well-shuffled deck

Outcome

- A possible result of an experiment or a trial.
- Examples:



Six outcomes for rolling a dice: 1, 2, 3, 4, 5, 6

Two outcomes for coin toss: Heads, Tails

Event

- A set of one or more outcomes for a random experiment.
- Example:
  - Getting a tail when a coin is tossed.
  - Getting an odd number when a dice is rolled.

2. Types of Probability

Types of Probability

- Experimental Probability
- Theoretical Probability

2.1 Theoretical Probability

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

When a coin is tossed:

- The probability of getting a head is $\frac{1}{2}$
- The probability of getting a tail is $\frac{1}{2}$

The probability $P(E)$ of an event will be a number such that,

$$0 \leq P(E) \leq 1$$

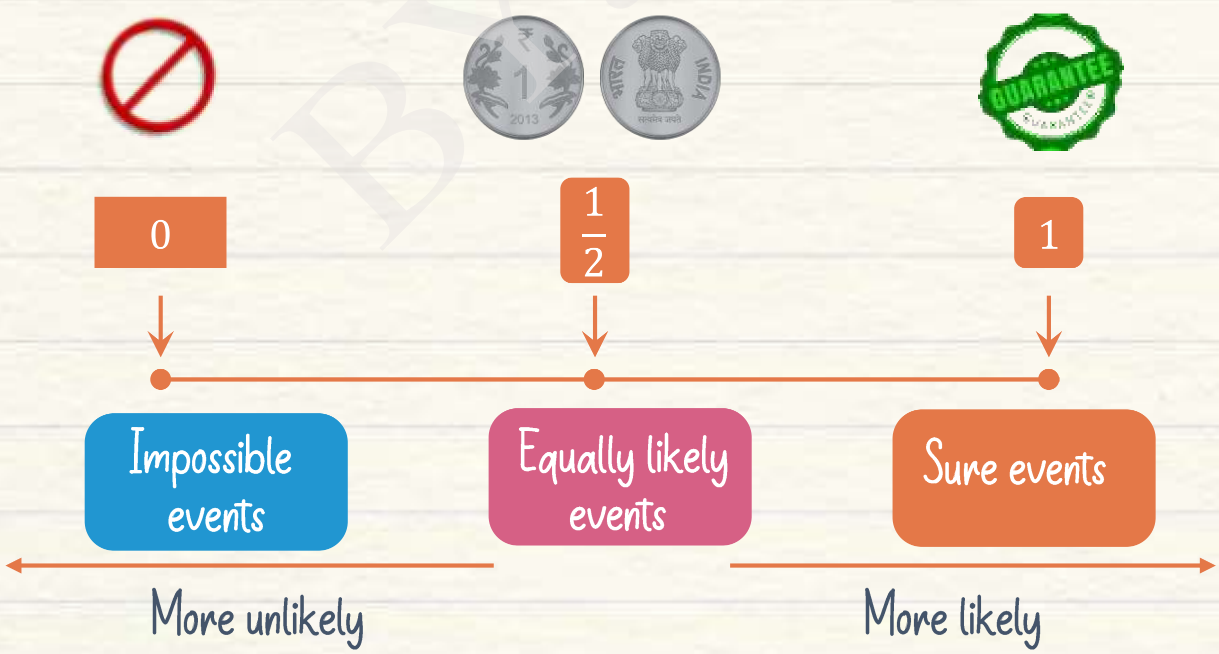
3. Types of Events

Elementary Event

- Has as only one outcome.
- Sum of all the elementary events for an experiment = 1

Equally likely Event

- When all the outcomes of an experiment have the same chance of occurring.
- Example: Tossing a coin



Impossible Event

- P(E) = 0.
- Example: Getting a 7 when rolling a die

Sure / Certain Event

- P(E) = 1.
- Example: Christmas being celebrated on the 25th of December

3. Types of Events

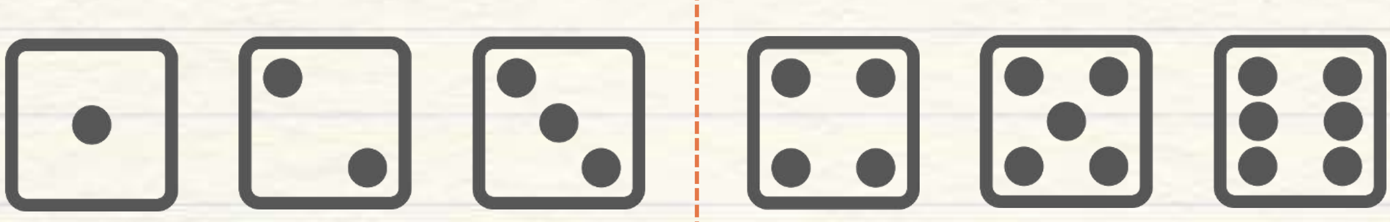
Complementary Events

- IF E denotes happening of an event, then $\bar{E}$ denotes NOT happening of that event.
- E and $\bar{E}$ are said to be complementary events.
- $\bar{E}$ is the complement of E.

$$P(\bar{E}) = 1 - P(E)$$

For an event of getting a number less than four on rolling a dice:

E	$\bar{E}$



4. Important Formulae

Theoretical Probability

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

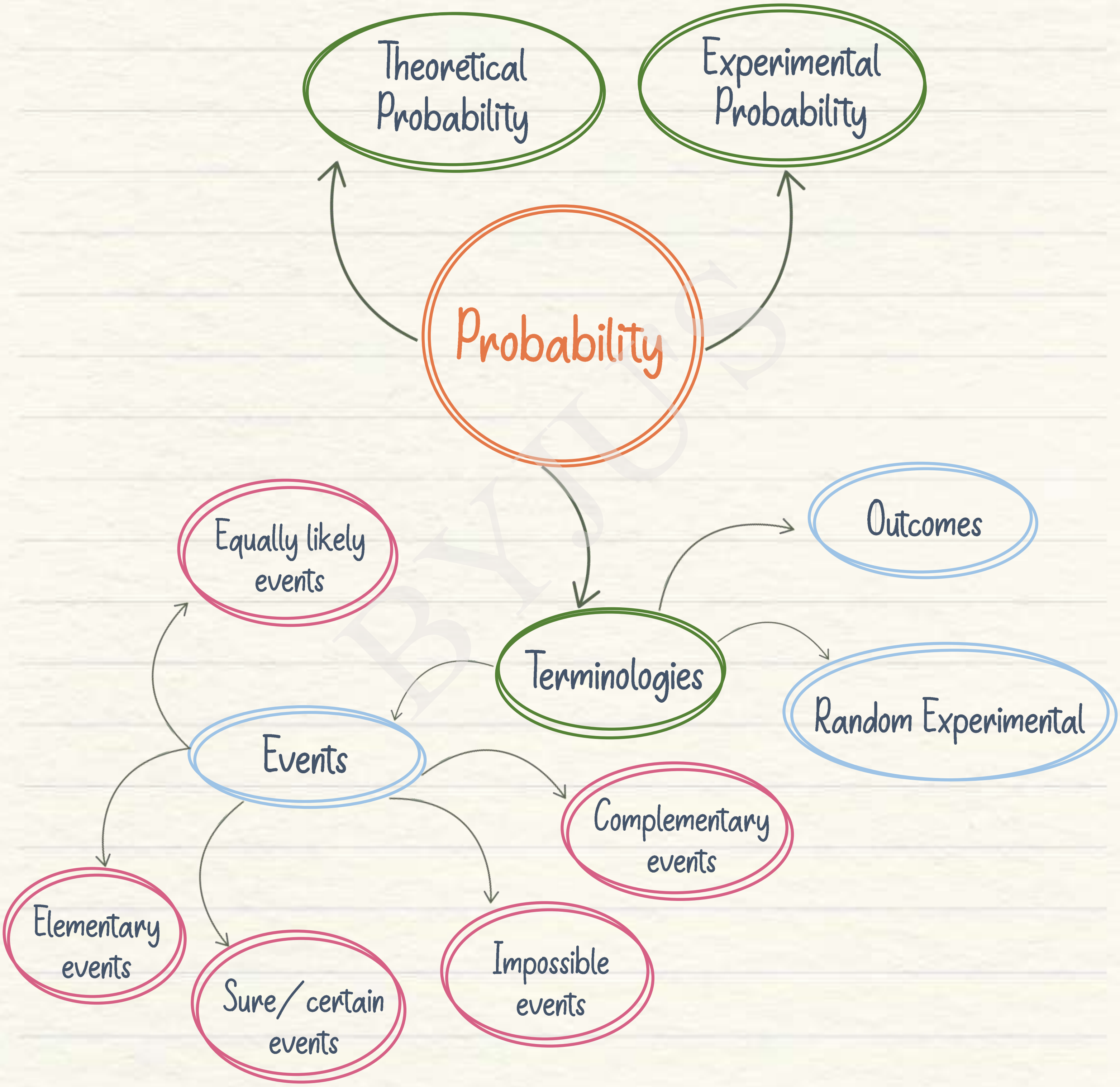
Probability of an event

$$0 \leq P(E) \leq 1$$

For two complementary events, E and $\bar{E}$,

$$P(\bar{E}) = 1 - P(E)$$

Mind Map


Probability
Theoretical Probability
Experimental Probability
Terminologies
Outcomes
Random Experimental
Events
Equally likely events
Complementary events
Elementary events
Sure/ certain events
Impossible events